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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

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A STUDY OF THE ULTRA-VIOLET END OF THE SOLAR SPECTRUM

By CHARLES FABRY AND H. BUISSON

ABSTRACT

Solar spectrum from λ 3150 to λ 2900 A.—To obtain photographs of this region it is necessary to eliminate diffuse light of longer wave-length. This was done by using a *double spectrograph* consisting of two quartz spectrographs in series arranged with their dispersion planes at right angles. The intensity of the spectrum falls off very rapidly, being only one-millionth as great at λ 2900 as at λ 3150; therefore, to enable the whole region to be photographed at once for measurements of absorption, *two absorption screens* made by intensifying photographic films with $HgCl_2$ were used in echelon to reduce the intensity above λ 2990. To obtain a *photographic map* with uniform normal exposure throughout, two plates, each covering half the region, were exposed by moving a shutter by hand at proper predetermined rates across each spectrum (see Plate III). The wave-lengths will be published soon.

Absorption of the atmosphere for λ 3150 to λ 2900 A.—By comparing the intensities of spectrograms taken for various zenith angles, the *coefficients* for various wave-lengths were determined. The effect of molecular diffusion was corrected for by using the Rayleigh formula, and the effect of haze was eliminated by assuming it independent of wave-length. A comparison with the corresponding coefficients for ozone, previously determined by the authors, shows remarkable agreement, leaving no doubt that this absorption is due to ozone.

Ozone in the atmosphere is found to be equivalent to a layer about 3 mm thick at atmospheric pressure. The *daily values* for May and June, 1920, vary irregularly from 0.29 to 0.34 cm. It would be interesting to follow these *variations* through a solar period. It is shown that the *location of this ozone* must be in the outer layers of the atmosphere, above 40 km, and in *explanation of its origin* it is suggested that the ozone is produced by solar rays of wave-length shorter than λ 2000 which penetrate only the outer layers and that longer rays dissociate the ozone and prevent its accumulation beyond the amount for equilibrium.

Distribution of energy in the solar spectrum outside our atmosphere for λ 3150 to λ 2900 A was determined by correcting the observed intensities for the absorption by the atmosphere and then reducing to units of energy by the use of a calibration-curve obtained by comparing the intensity-curve of an arc with the radiation from a black body at 3750° Abs. The energy-curve thus obtained shows only a gradual slope

toward the shorter wave-lengths. For λλ 3143, 2997, 2922, and 2898 the relative energies for the sun outside the earth's atmosphere come out 155,000, 218,000, 145,000, and 45,000 respectively, while at the surface of the earth when the sun is at the zenith the corresponding values are about 22,400, 1320, 2.2, and 0.02.

1. *Introduction.*—The very rapid falling off in the intensity of the solar spectrum in the ultra-violet region, which is without any doubt due to the absorption by the earth's atmosphere, has been attributed to one of the constituents of that atmosphere, ozone.[1] This seems in fact to be the only gas having absorptive properties which would permit of an explanation of the observed phenomena. Quantitative determinations of the intensity in this spectral region are completely lacking, however, although they are very numerous in the visible and in the infra-red spectrum. In the absence of such measures, the explanation of this cutting off of the spectrum as an effect of ozone is a trifle hypothetical, and we do not know the amount of that gas which intervenes between the sun and us. Further, we have no information as to the intensity of the solar radiations of that short wave-length prior to their entry into our atmosphere.

We have undertaken to make numerical determinations of the intensity, to measure the coefficients of absorption of the atmosphere for each wave-length, and to see if those coefficients accord with those of ozone. If so, we shall then be able to determine the quantity of ozone which produces the absorption. Finally, on correcting each radiation for the absorption which it has suffered, we shall be able to learn the intensity outside the atmosphere and to see if the spectrum thus freed from terrestrial absorption still shows a marked weakening in that region. Should this be the case, it would be necessary to conclude that solar phenomena have a part in this limitation of the spectrum.

Our measures were undertaken long ago, beginning with the study of the absorption of ozone in 1912.[2] It was necessary to make this study because its absorption was little known, whence it was impossible to compare it with that of the atmosphere.

[1] The presence of ozone in the atmosphere has recently been established beyond doubt by Fowler and Strutt, who have shown the existence of absorption bands of this gas in the region λ 3300, in the light of the sun and of the stars. *Proceedings of the Royal Society*, A, **93**, 577, 1917.

[2] *Journal de Physique* (5), **3**, 196, 1913.

The first attempts to measure the atmospheric absorption were made at that time and were continued in the course of the following years until interrupted by the war; they were resumed during the last two years. It is unnecessary to describe the succession of improvements, and we limit ourselves to describing the actual arrangement of apparatus and methods of calculation which we finally adopted.

Our experiments were made at the Faculty of Sciences, of the University of Marseille (latitude $43°17'$, altitude 25 meters), under conditions which are not the most favorable, in the center of an industrial city where the air is seldom pure.

2. *Methods.*—The region of spectrum which we have studied is of shorter wave-length than λ 3150, and we have employed solely the photographic method for the measures of intensity. We could not dream of using a thermopile or a bolometer, which would have had the advantage of giving intensities directly in energy; but the region to be measured is greatly weakened by absorption, so that the radiation at λ 2900 is reduced, even for the sun at the zenith, to at least 10^{-5} of its value outside of the atmosphere.

The problem to be solved, then, is the comparison of the intensity of one and the same radiation at different hours of the day, which is relatively easy because it is free from all the difficulties due to the selective effect on different rays. On the same plate and with the same length of exposure, exposures are made at different hours. When the plate is developed, the blackening is more intense as the intensity of the effective radiation is greater. It is possible to graduate the plate as to intensity, that is to say, to establish the relation existing between the blackening and the intensity by taking two exposures which should be of known ratio. The difference of density of those two exposures, divided by the logarithm of the ratio of the intensities, gives the ratio γ which defines, for that plate, the relation sought in the region of normal exposure.

To make two exposures in known ratios of intensity, we employed two different procedures: in the first, we placed in the path of the luminous beam a polarizer and an analyzer, the latter being fixed while the polarizer was mounted on a graduated circle so that it

could be differently oriented. The beam was thus weakened in any desired ratio. The polarizers employed were Foucault prisms with an air space. It was in fact necessary to avoid apparatus with a layer of balsam, which is opaque to ultra-violet rays. This procedure grades the light in a direct manner, but it causes a somewhat complicated arrangement of apparatus; further, in our regular measures we have preferred to work with absorption. Filters, the construction and use of which will be described below, may be interposed in the beam. Their opacity having been measured once for all for each radiation by a polarimetric method, they then furnish a simple and convenient means of graduating the light.

All of the preceding applies to the intensity of one and the same ray. We may then go farther, and give, in the short spectral region which we have studied (λ 3150 to λ 2900), values of the photographic intensities of the different radiations expressed in the same unit. Such a plan would be meaningless if we wished to apply it to the whole of the spectrum. The law of opacity always keeping the same general form, is then very different from one ray to another in consequence of the great variation of the ratio γ, which may increase from one to two in passing from the region λ 3000 to the visible region. Two rays which give identical opacities will cease to do so, if their intensities are increased tenfold. This difficulty does not exist in our case, at least with the plates which we have employed. The factor γ remains constant during the interval studied, and consequently the ratios of photographic intensities have a value as well defined for two different rays as for the same ray. It is true that these photographic intensities thus measured for neighboring radiations should not be confused with the true intensities expressed in energy: they differ according to the law of dispersion of the spectroscopic apparatus and according to the variable sensitiveness of the plate. The essential thing is that they have a definite significance. We shall see farther on how we may pass from photographic intensities to intensities of energy.

All these determinations depend on the measures of density at different parts of the photographic plates. These densities

ought to be measured on regions of very small extent, limited by the very narrow intervals which separate the solar lines. It was with such measures in view that we have constructed our microphotometer, which is particularly well adapted to this class of work.[1]

We have employed almost solely the photographic plates of Jougla, bromo-iodide of silver (mauve label), which have great sensitiveness and a normal range of exposure.

3. *Spectroscope.*—In photographing the extreme end of the solar spectrum with an ordinary spectroscope, it is immediately evident that it becomes lost in an intense veil which limits the observable region. This veil is produced by the diffused light due to the more intense radiations of the spectrum, the diffusion itself being due to the passage of the light through the different transparent media of the dispersive apparatus. It is absolutely necessary to eliminate this foreign light of longer wave-length from the region to be studied. The use of absorbing filters serves this problem only very incompletely; a preferable arrangement is to employ a second dispersion which throws to one side of the spectral band the foreign light which contaminates it. This procedure has already been employed in an imperfect way by Miethe and Lehmann,[2] who interposed, at some distance from the principal prism, a second prism having its edge perpendicular to the first.

Our arrangement is as follows. The first spectroscope, having prisms and lenses of quartz, produces a real spectrum, one portion of which, limited in length and height, is isolated from the rest by a slit placed at a convenient place. A second quartz spectroscope disperses, in a plane perpendicular to the plane of dispersion of the first spectroscope, the light which has passed through that slit; thus the diaphragm, which has limited the first spectrum, forms the slit of the second spectroscope. The whole apparatus is shown in perspective in Figure 1; the collimator and tube of the first spectroscope are in a horizontal plane with the edges of the prisms P and P' vertical. The quartz objectives L and L' are each of focal length 1 meter and diameter 58 mm. The two quartz prisms

[1] *Journal de Physique* (5), **9**, 37, 1919.

[2] *Sitzungsberichte der preussische Akademie der Wissenschaften*, **1**, 268, 1909.

are traversed by the beam along their optical axes; the one is right-handed, the other left-handed. They have a height of 50 mm, and their base is an equilateral triangle of 57 mm side. We allow an amount of light that is strictly necessary to enter the apparatus, by limiting the height utilized of the slit F by means of a screen having an opening 1 mm in height. The spectrum produced at S is spread out horizontally and it is received on a diaphragm which limits it to the region between λ 3200 and λ 2800. The aperture of that diaphragm is about 40 mm long and 1 mm high. The second

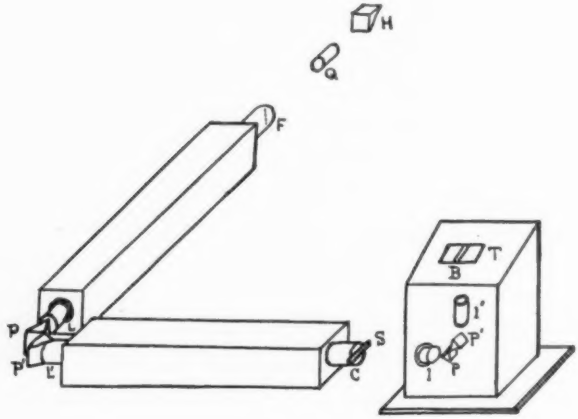


FIG. 1

spectroscope, which is arranged to disperse the light in a vertical plane, used that aperture as its slit, and is produced by two equilateral prisms of quartz p and p', the edges of which are horizontal, together with two lenses l and l', of focal lengths 25 and 17 cm. Another lens, C, of focal length 30 cm, placed in the plane of the slit S, plays the rôle of field lens, projecting an image of the prisms P of the first spectroscope upon those of the prisms p of the second spectroscope, so that all the useful light which has passed through the first apparatus may be gathered by the second.

If the slit S of the diaphragm was illuminated by a monochromatic light, the second spectroscope would give a straight line as an image; but since the different parts of that slit correspond to different wave-lengths, there is produced by dispersion an image

of which the points are unequally deviated, or, in other words, a curved image is produced. The foreign rays of less refrangibility, which form the diffuse light, are deviated completely beyond that image, which then is seen on a background entirely free from veil.

As neither of these two spectroscopes is achromatic, the final spectrum must occupy a complicated position in space. The position and the inclination which must be given to the photographic plate *T* have been determined by a series of preliminary experiments; but in spite of the complication of the optical path, the spectrum is sufficiently sharp. The scale of the spectrum is about 1 mm for 10 angstroms, and thus it is possible to separate two rays having wave-lengths differing by at least half an angstrom. We have thus obtained completely the object sought, and it is possible to make exposures of a quarter of an hour on the solar spectrum without having the extremity of the spectrum limited by the fog, and thus it is possible to push the study of the spectrum out to its actual limit.

This whole arrangement, formed by the two spectroscopes thus placed in tandem, forms an entirely rigid system, each part of which has been placed in its definitive position. The quartz parts were cut by M. Jobin and the remainder of the equipment was assembled in the workshop of the laboratory.

A heliostat, having a totally reflecting prism *H*, sends the solar beam into the apparatus with but a single reflection, and an achromatic lens *Q*, of quartz and fluorite, of 30 cm focal length, projects an image of the sun on the slit.

4. *Filters.*—The variation in the intensity in the region of the spectrum studied is so great that a single exposure of a given length does not furnish a plate that can be used over more than an extremely short interval. The regions of long wave-length are overexposed, while those of shorter wave-length need an excessive increase of exposure. In order to make the photographic impression more uniform from one exposure and thus to be able to extend the measures made on a single plate, we have greatly reduced the intensity for longer wave-lengths by the following device. Over a portion of the slit *S*, where the horizontal spectrum from the first spectroscope is projected, we place absorbing

screens of measured opacity, which reduce in a known ratio the intensity of the rays which pass through them. Thus we weaken the rays that are too intense, while allowing the others to pass freely, and we thus obtain on the plate a spectrum covering several regions which have all received a suitable exposure. With the judicious choice of filters, we thus obtain a spectrum throughout the whole length of which measures of intensity are possible.

After several attempts, we made these filters by taking gelatine films from photographic plates which had been developed and fixed and had been separated from the glass. Thus we had absorbers of very small thickness, of which the opacity could be selected at will. The coatings of ordinary photographic plates have one very serious inconvenience, for they are made by grains of silver scattered through the gelatine, and also have the anomaly of the transparence of the metallic silver in the region of λ 3100, so that they have there very slight opacity. Thus we should have filters of which the opacity would be very variable with the wave-length, and this in just the region which interests us.

We have gotten entirely rid of this disturbing property by intensifying the plate with mercuric chloride and ammonia before removing the gelatine. The layers thus obtained have an absorption without anomalies, which increases gently and uniformly as the wave-length decreases, but so that this variation is not disturbing to our measures. With these filters, we made the following arrangement: a sheet of cardboard contains an aperture on which was placed a strip of gelatine, detached from a plate, so that it should cover only a part of the aperture; then a second strip of gelatine was placed over a part of the first one, thus giving two successive echelons. The supporting cardboard is always placed in the same way before the slit S upon which the first spectrum is projected. Under this condition the region beyond λ 2990 does not suffer any absorption, while that between λ 2990 and λ 3040 undergoes absorption from a single strip of gelatine, and finally the region of wave-lengths greater than λ 3040 suffers absorption from two superposed strips.

We have determined the absorption of these screens under the conditions under which we use them, namely, in the apparatus

where they are to be used and not by measurements made under other conditions. The passage of light through these photographic films is accompanied by a slight amount of diffusion and the impression produced on the plate varies with the proportion of the diffused light used in forming the image. It is therefore necessary to make the measures of absorption upon the filter when in the place where it is going to be used and without any modification of the aperture of the luminous beams.

The transmission of the filters was measured by the polarimetric method described further on and was made separately on each of

TABLE I

LOGARITHMS OF OPACITIES OF FILTERS

λ	Filters		
	A	*E*	*C+E*
2946	1.12	....	....
2956	1.11	....	....
2963	1.10	....	....
2997	1.08	2.08	....
3022	1.06	2.01	....
3052	1.04	....	3.35
3104	1.02	....	3.33
3143	1.01	....	3.31

the two filters which were used on the spectrum. One of these transmits about 1/100 of the incident light in the region of λ 3000, while the other transmits about 4 per cent; when they are superposed, the transparence is not over 0.0004.

It was further necessary to calibrate each plate on which the solar spectrum was photographed. This was accomplished by interposing for the whole of the beam, before its entry into the apparatus, a filter of the same sort as those described above, which weakened all the radiations in a known ratio.

The results of these measures are given in Table I, containing the logarithms of the opacities of the different filters. Filter *A* is the one which weakened the spectrum for calibrating the plate. Filters *C* and *E* are the ones placed over the slit *S* in the focal plane of the first spectroscope; *E* is introduced only in the region λ 3000; for wave-lengths greater than λ 3040 the light traverses the two filters *C* and *E*.

5. *Determination of the coefficient of absorption of the atmosphere for the different rays.*—We have followed the method proposed in principle by Bouguer two centuries ago, which has been correctly applied to monochromatic radiations from the sun by Langley and afterward by Abbot. This consists in studying the decrease of intensity of a definite ray when the zenith distance of the sun is increasing. If we let I_0 be the intensity of a ray before its entry into our atmosphere, the sun being at the zenith, it will arrive at the surface of the earth with a reduced intensity I_1 and we may define the coefficient of absorption of the atmosphere for that ray by the equation

$$m = \log_{10} (I_0/I_1). \tag{1}$$

Now if the sun is at zenith distance z, the path traversed by the light in each stratum is multiplied by sec z, and the intensity I which reaches the earth's surface is given by the formula

$$\log I = \log I_0 - m \sec z. \tag{2}$$

For determining the coefficient m we measure the intensity at different hours of the day and then trace a curve having for abscissas the values of sec z and for ordinates the values of log I. If the transparence of the air has not undergone any change during the course of the observation, this curve will be a straight line, the slope of which gives us the value of m. The solar spectrum is photographed at different hours of the day, on the same plate, with equal times of exposure, the filters C and E, mentioned above, being interposed to weaken the portions that are too strong. An identical exposure is further made at noon through the filter A to obtain the necessary data as to the calibration of the plate. We then measure with the microphotometer the density of the plate on the two spectra for rays conveniently chosen. In this choice we are limited by the great number of dark lines which occur in the solar spectrum in that region. Among these rays we have selected eleven intervals where the spectrum has a very uniform appearance, from λ 2922 to λ 3143. All of our measures refer to the points thus determined.

Thanks to the precaution taken in rendering the spectrum uniform by means of the absorbing filters, we have been able to keep

the blackening of the plate in the region of normal exposure, which is that where the curve connecting the photographic densities with the logarithm of the intensity acting is a straight line. The slope of that right line (coefficient γ) independent of the wave-length in the short spectral region studied here, is obtained by two exposures made at noon, one under general conditions and the other with a known reduction of brightness. We then obtain for every ray the intensity at different hours and further the photographic intensities of the different radiations.

TABLE II

Local Solar Time	λ 2922	λ 2931	λ 2936	λ 2946	λ 2956	λ 2963	λ 2997	λ 3022	λ 3052	λ 3104	λ 3143
8^h22^m	.....	.....	.....	.....	0.66	1.23	0.67	1.56	0.96	1.77	2.14
8 39	.....	.....	.....	0.28	1.10	1.61	0.96	1.85	1.12	1.91	2.24
8 56	.....	.....	.....	0.63	1.44	2.01	1.17	2.05	1.25	2.04	2.35
9 16	.....	.....	0.38	1.05	1.86	2.28	1.47	2.22	1.46	2.09	2.45
9 38	.....	0.35	0.74	1.44	2.20	2.64	1.72	2.41	1.58	2.18	2.59
10 2	.....	0.65	1.05	1.72	2.46	2.95	1.87	2.54	1.64	2.27	2.63
10 31	0.28	0.84	1.28	1.91	2.64	3.10	1.94	2.64	1.70	2.31	2.65
11 25	0.52	1.10	1.55	2.15	2.92	3.27	2.10	2.78	1.78	2.37	2.68
13 22	0.37	0.97	1.39	2.03	2.77	3.23	2.03	2.71	1.76	2.39	2.68
13 52	.....	0.68	1.06	1.75	2.50	2.95	1.89	2.57	1.67	2.28	2.60
14 24	.....	0.37	0.72	1.43	2.20	2.72	1.72	2.45	1.62	2.27	2.56
14 44	.....	.....	0.46	1.12	1.92	2.52	1.52	2.27	1.50	2.17	2.47
15 4	.....	.....	.....	0.79	1.64	2.07	1.35	2.13	1.37	2.13	2.41
15 20	.....	.....	.....	0.44	1.28	1.83	1.13	2.01	1.25	2.01	2.34
15 46	.....	.....	.....	.....	0.71	1.19	0.76	1.68	1.03	1.86	2.16
11 23*	.....	.....	.....	0.62	1.33	1.69	0.58	1.22	.....	0.98	1.29

* With filter *A*.

6. *Example of a day's observations.*—We shall give as an example the measures made on June 7, 1920. Fifteen exposures, each of one minute, were spread over a total time of seven and one-half hours; an exposure of the same duration through the filter *A* was taken for the purpose of calibrating the plate. The densities of the plate for each exposure and for each of the selected rays are given in the Table II where the first column contains the hour of the different exposures in local solar time, while each of the others refers to one of the rays measured. A mere examination of Table II shows how the intensity decreases rapidly when the sun departs from the zenith, and how the effect is more marked for the short wave-lengths. We further see the useful effect of the

absorbing filters placed in the spectra; the measures would be impossible on account of the overexposure involved for wave-lengths longer than 2963 if the intensity had not been reduced to a convenient value by the presence of the filters. The last line of the table refers to the exposure made at 11^h23^m through the filter *A;* from that comparison with the exposure made without absorption at 11^h25^m, for determining the calibration of the plate, we have adopted the number 1.40 as the value of the coefficient γ, this being the mean of values differing very slightly for different radiations. On dividing the densities given in the preceding table by that figure, we obtain the intensities of the different rays which have produced the blackening. For those which have been weakened by the interposition of the filter in the spectrum, it is necessary to take account of that absorption by adding the densities of the filter given in Table I. We thus obtain Table III, which gives in logarithms the values of the photographic intensity of each ray at different hours. The first column of that table contains the hours and the second the secant of the zenith distance of the sun. These measures must be corrected for two causes of weakening, from which all radiations suffer, in addition to the very selective absorption for which we are here seeking the origin. These two causes are the diffusion by the molecules of the air and the extinction produced by the haze or other foreign bodies in suspension in the air. The molecular diffusion obeys laws which are known, and its effect may be calculated a priori; it is enough to know its effect on each ray when the sun is at the zenith. Table IV gives the values of the coefficient β defined by

$$\beta = \log (I_0/I_1),$$

in which I_0 is the intensity outside the atmosphere and I_1 is the intensity which would be found at the earth's surface if diffusion by the molecules was the only effect existing. These data have been calculated by Lord Rayleigh's formula.

We have eliminated the effect of absorption by foreign particles, which may be very variable from moment to moment, in assuming that this action was subject to little variation as a function of the

wave-length in the narrow spectral region covered by our measures. The method of computation is as follows.

Let m be the coefficient of absorption of the atmosphere for vertical transmission, due to that unknown substance which acts selectively, and let β be the coefficient of extinction due to molecular diffusion under the same conditions. These two quantities depend

TABLE III

Time	Sec z	λ 2922	λ 2931	λ 2936	λ 2946	λ 2956	λ 2963	λ 2997	λ 3022	λ 3052	λ 3104	λ 3143
8^h22^m....	1.526	.....	.....	.....	.....	0.47	0.88	2.56	3.12	4.04	4.59	4.84
8 39	1.440	.....	.....	.....	0.20	0.78	1.15	2.76	3.33	4.15	4.69	4.91
8 56	1.367	.....	.....	.....	0.45	1.03	1.43	2.91	3.47	4.24	4.79	4.99
9 16	1.295	.....	.....	0.27	0.75	1.33	1.63	3.13	3.60	4.39	4.82	5.06
9 38	1.232	.....	0.25	0.53	1.03	1.57	1.88	3.31	3.73	4.48	4.89	5.16
10 2	1.177	.....	0.46	0.75	1.23	1.76	2.11	3.41	3.82	4.52	4.95	5.19
10 31	1.128	0.20	0.60	0.91	1.36	1.88	2.21	3.46	3.90	4.56	4.98	5.20
11 25	1.077	0.37	0.78	1.11	1.53	2.08	2.33	3.58	4.00	4.62	5.02	5.22
13 22	1.118	0.26	0.69	0.99	1.45	1.98	2.31	3.53	3.95	4.61	5.04	5.22
13 52	1.165	.....	0.48	0.76	1.25	1.78	2.11	3.43	3.85	4.54	4.96	5.17
14 24	1.237	.....	0.26	0.51	1.02	1.57	1.94	3.31	3.76	4.51	4.95	5.14
14 44	1.295	.....	.....	0.33	0.80	1.37	1.80	3.16	3.63	4.42	4.88	5.07
15 4	1.367	.....	.....	.....	0.56	1.17	1.48	3.04	3.53	4.33	4.85	5.03
15 20	1.435	.....	.....	.....	0.31	0.91	1.31	2.89	3.45	4.24	4.77	4.98
15 46	1.573	.....	.....	.....	.....	0.51	0.85	2.62	3.21	4.08	4.66	4.85

TABLE IV

λ............	2922	2931	2936	2946	2956	2963	2997	3022	3052	3104	3143
β............	0.48	0.47	0.46	0.46	0.45	0.45	0.43	0.42	0.40	0.38	0.36

only on the wave-length. If I_0 is the intensity outside the atmosphere, the intensity I at the surface of the earth will be given by the equation

$$\log I = \log I_0 - (m+\beta) \sec z - \delta, \tag{3}$$

in which z is the zenith distance of the sun and δ represents the effect of the haze, irregularly variable from moment to moment, but independent of the wave-length. At the same moment for another ray, we have the analogous relation

$$\log I' = \log I'_0 - (m'+\beta') \sec z - \delta, \tag{3'}$$

whence we have

$$\log I - \log I' = K - (m - m' + \beta - \beta') \sec z, \tag{4}$$

in which K is a quantity independent of the time. If we compare each of the rays with one of them, for example with λ 3143, the difference of the logarithms of the intensities of the radiations λ and 3143 should vary linearly as a function of sec z, and the slope of the right line which represents that variation gives for each ray the value of

$$m-m'+\beta-\beta'.$$

Let us now form the table of the differences log I − log I', understanding that I' refers to λ 3143. To avoid negative numbers,

TABLE V

Sec z	λ 2922	λ 2931	λ 2936	λ 2946	λ 2956	λ 2963	λ 2997	λ 3022	λ 3052	λ 3104	λ 3143
1.526	.....	.....	.....	.....	0.85	1.26	2.94	3.50	4.42	4.97	5.22
1.440	.....	.....	.....	0.51	1.09	1.46	3.07	3.64	4.46	5.00	5.22
1.367	.....	.....	.....	0.68	1.26	1.66	3.14	3.70	4.47	5.02	5.22
1.295	.....	.....	0.43	0.91	1.49	1.79	3.29	3.76	4.55	4.98	5.22
1.232	.....	0.31	0.59	1.09	1.63	1.94	3.37	3.79	4.54	4.95	5.22
1.177	.....	0.49	0.78	1.26	1.79	2.14	3.44	3.85	4.55	4.98	5.22
1.128	0.22	0.62	0.93	1.38	1.90	2.23	3.48	3.92	4.58	5.00	5.22
1.077	0.37	0.78	1.11	1.53	2.08	2.33	3.58	4.00	4.62	5.02	5.22
1.118	0.26	0.69	0.99	1.45	1.98	2.31	3.53	3.95	4.61	5.04	5.22
1.165	.....	0.53	0.81	1.30	1.83	2.16	3.48	3.90	4.59	5.01	5.22
1.237	.....	0.34	0.59	1.10	1.65	2.02	3.39	3.84	4.59	5.03	5.22
1.295	.....	.....	0.48	0.95	1.52	1.95	3.31	3.78	4.57	5.03	5.22
1.367	.....	.....	.....	0.75	1.36	1.67	3.23	3.72	4.52	5.04	5.22
1.435	.....	.....	.....	0.55	1.15	1.55	3.13	3.69	4.48	5.01	5.22
1.573	.....	.....	.....	.....	0.88	1.22	2.99	3.58	4.45	5.03	5.22

all the differences have been increased by the constant quantity 5.22, which amounts to making an arbitrary choice of the value K in the preceding formula. Thus we obtain Table V, which gives the logarithms of the intensities of the different rays at different hours referred to what they would be if λ 3143 did not suffer any absorption. If we trace for each ray the curve of the numbers given in each column as a function of sec z, we obtain the graph in Figure 2, on which the + sign represents the points referring to measures in the morning, and the symbol × to measures in the afternoon. The slope of each of these straight lines gives the values of

$$m-m'+\beta-\beta'.$$

In correcting for the values $\beta-\beta'$, known from Table IV, there remain the values of $m-m'$, which are the differences of the

coefficient of absorption of the atmosphere for the ray λ and the ray 3143.

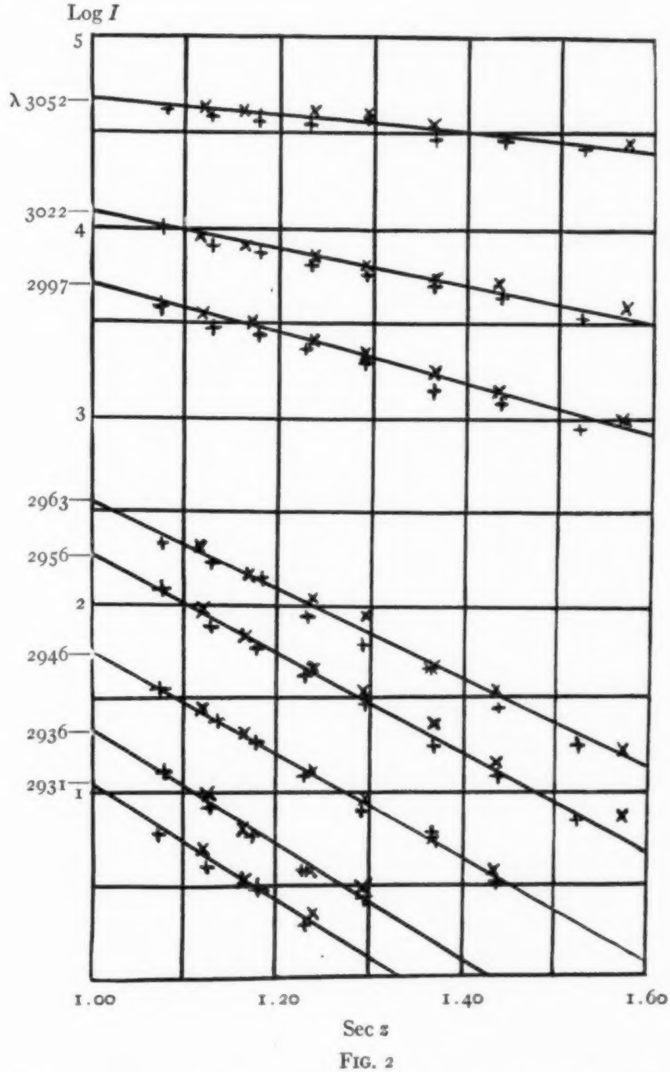


FIG. 2

It is necessary to compare these values with the coefficients of absorption of substances which are supposed to cause that absorption. If they are functions of wave-length, according to the same

law of variation as these coefficients, we may affirm that the substance in question is the one present in the atmosphere and which limits the solar spectrum. If a is the coefficient of absorption of the substance considered, and x its thickness, we ought to have

$$m = ax,$$
$$m' = a'x,$$
$$m - m' = (a - a')x.$$

Table VI gives for the wave-lengths selected the values of the coefficient β, the differences $\beta - \beta'$, the coefficient a of the ozone, the differences $a - a'$, the slopes P measured on the graph, the values of $m - m'$, and finally the quotients respectively of $m - m'$ divided by $a - a'$, which ought to yield the value of x, which is the

TABLE VI

λ	β	$\beta-\beta'$	a	$a-a'$	P	$m-m'$	x
2931	0.47	0.11	11.2	10.5	3.15	3.04	0.290
2936	0.46	0.10	10.5	9.8	3.16	3.06	0.312
2946	0.46	0.10	9.3	8.6	2.78	2.68	0.312
2956	0.45	0.09	8.1	7.4	2.66	2.57	0.347
2963	0.45	0.09	7.4	6.7	2.37	2.28	0.340
2997	0.43	0.07	4.7	4.0	1.32	1.25	0.309
3022	0.42	0.06	3.4	2.7	0.96	0.90	0.333
3052	0.40	0.04	2.3	1.6	0.44	0.40	0.250
3143	0.36	0.00	0.7	0	0	0	....

thickness in cm of the stratum of ozone. The rays λ 2922 and λ 3104 have not been utilized, the first because the corresponding measures have not been sufficiently accurate, and the second because its slope is too small. Finally, it should be recalled that λ 3143 is the comparison ray. We see that the values of x obtained for the different rays are sensibly equal; the agreement is still better if we except the two extremes, which are less accurate than the others. The conclusion is forced upon us that it is the ozone which limits the solar spectrum by its absorption.

The adopted mean of the values in the last column, 0.325, gives for June 7, 1920, the thickness of pure ozone expressed in cm, at atmospheric pressure, which would be traversed by the solar rays were the sun at the zenith. Table VII summarizes the measures made by the same method on three different days. We see that the accordance between the different values of x for

each day is satisfactory, whence it follows that the quantity of ozone present in the atmosphere was sensibly the same for these three days. But this is not always the case, and we have established beyond doubt the existence of considerable variation in the amount of ozone from one day to another. We shall return to this question farther on.

7. *Mode of calculation.*—The method of computation which we have employed was necessary to eliminate the irregularities of absorption caused by the dust, the haze and the smoke, varying from one moment to the next, but affecting in the same manner all the rays in the short region which we have measured. If

TABLE VII

λ	May 13	May 26	June 7
2931	....	0.34	0.29
2936	0.30	0.34	0.31
2946	0.33	0.32	0.31
2956	0.33	0.33	0.35
2963	0.35	0.35	0.34
2997	0.33	0.36	0.31
3022	0.38	0.34	0.33
3052	0.30	0.33	0.25
Adopted value	0.33	0.34	0.32

the purity of the atmosphere was constant during the whole day, one could immediately apply the method of secants to each ray and thus obtain the total absorption of the atmosphere. This is the case for the very beautiful days when the sky is exceptionally pure. This may be recognized on the diagram giving the intensity as a function of sec z, by the fact that the points then fall on a straight line. This happened on June 7, 1920, as shown by the graph for the ray 3143 (Fig. 3). The slope of the right line on which fall all the points both of the morning and of the afternoon, gives the value 0.84 for the logarithm of the opacity for that ray of the atmosphere traversed vertically. In subtracting from that number the part referring to the ozone, 0.23, and the part due to the molecular diffusion, 0.36, we may deduce the effect of the haze alone; we thus find that the logarithm of its opacity is 0.25 for vertical rays.

We do not find the same regularity for most of the days of observation. The points do not fall on a straight line and in particular the observations made in the morning do not accord with those made in the afternoon. This renders impossible any precise determination of the total opacity, which is so variable in the course of the day; but the method of calculation which we have employed is general and causes these irregularities to disappear.

8. *The solar spectrum freed from atmospheric absorption.*—It is now possible to calculate the intensities which the solar rays would have if they were not weakened by our atmosphere. Equation 3, solved with respect to I_0, gives, without the necessity of

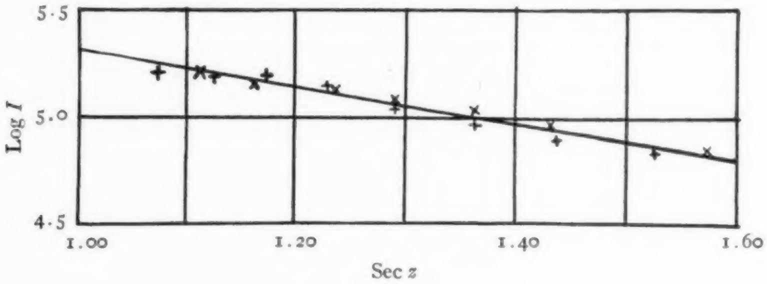


FIG. 3

introducing the value δ, the logarithms of the photographic intensities of the different rays expressed in a single arbitrary unit.

This computation has been made for those days of observation referred to above, by utilizing solely the exposure made near noon for which the term in sec z is the smallest. The results obtained are summarized in Table VIII. The measures stop at wavelength 2922, the shortest which we could obtain with an exposure of one minute. It is possible to extend the spectrum by increasing the length of exposure, at least when the sun is near the zenith, but the direct measures of the coefficient of absorption become impossible. These coefficients may be computed if a determination of the quantity of ozone has been made on the same day by the method which we will now indicate. We may then calculate the intensity before entrance into the atmosphere. This measurement has been made with exposures of 7^m and of 75^m on a plate to which had also

been given the usual exposure of one minute.[1] This time we have the necessary elements for determining the law of blackening as a function of the exposure time, and to refer to what we should have

TABLE VIII

LOGARITHMS OF THE PHOTOGRAPHIC INTENSITIES OUTSIDE THE ATMOSPHERE

	2922	2931	2936	2946	2956	2963	2997	3022	3052	3104	3143
May 13	.....	.....	5.35	5.36	5.41	5.47	5.69	5.55	5.86	5.64	5.71
May 26	.....	5.30	5.31	5.31	5.44	5.52	5.64	5.52	5.86	5.70	5.77
June 7	5.30	5.21	5.28	5.28	5.39	5.30	5.60	5.63	5.85	5.85	5.85
Mean	5.30	5.25	5.31	5.31	5.41	5.43	5.64	5.57	5.85	5.73	5.78

TABLE IX

	λ 2898	λ 2906	λ 2912	λ 2917
Log I	−1.57	−1.20	−0.45	+0.03
Log I_0	4.76	4.52	5.00	5.16

with the same length of exposure. Thus we arrive at a measurement of the intensity out as far as wave-length 2898. Table IX, based on the preceding one, gives the values of log I, the photographic intensity at the surface of the earth when the sun is at the zenith, and of log I_0, the photographic intensity outside the atmosphere. All of these measures were made on June 20, 1920.

9. *Calculation of the intensity expressed as energy.*—All the preceding numbers are relative to the photographic intensities which we have defined above. They differ from the true values of the intensity expressed in units of energy because the dispersive apparatus, not having a constant dispersion, decreases the blackening for the shorter wave-lengths and also because the sensitiveness of the plate is not the same for all the rays studied.

We have eliminated at one stroke the effect of the two causes by studying, with the same spectroscope and the same plates, the law of blackening in the spectrum of a source giving a continuous spectrum with a known distribution of energy. For this purpose

[1] On the exposure of 75 minutes, the last trace of photographic impression corresponds to the wave-length 2885.

we used the light from a carbon arc, the image of which was projected on the slit of the spectroscope, and we interposed a polarizer and an analyzer so that we could vary all the rays at the same time in any desired proportion. Operating as in the case of the sun, we constructed the curve of photographic intensity with the spectra of the arc. The curve A of the graph (Fig. 4) was thus obtained, the abscissas being the wave-lengths, and the ordinates the logarithms of the photographic intensities.

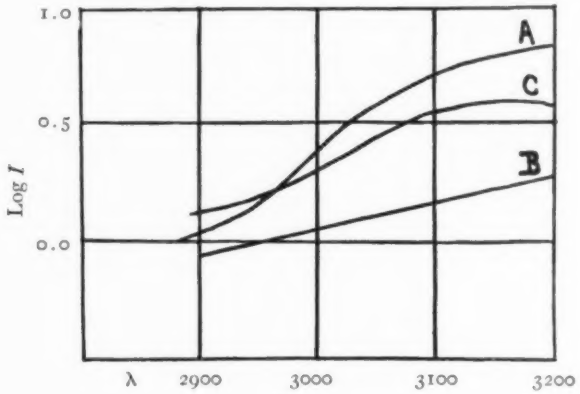


FIG. 4

Furthermore, the radiation of the crater of the arc is that of a black body at 3750° Abs. Its energy-curve as a function of the wave-length is known, being given by the equation

$$\log I = -5 \log \lambda - \frac{1.673}{\lambda} .$$

It is shown by the curve B of Figure 4. The difference of the ordinates of the curves A and B gives the value of the logarithm of the photographic intensity, for equal energy, for each wave-length; it is shown by curve C. If we now subtract the ordinates of curve C from the logarithms of the solar photographic intensities obtained before, we find for each ray the intensity that it would have in a normal spectrum expressed in units of energy. All of our results are summarized in Table X, which contains the values, already given, of the logarithms of the photographic intensities

of the rays before entering into the atmosphere, and then in logarithms and in numbers the intensity in energy of the same rays outside the atmosphere; furthermore, the logarithms of the total opacity of the atmosphere on the seventh of June for vertical rays; and finally the values as logarithms and numbers of the intensity in energy at the surface of the earth. The numbers in the last column show the extraordinary decrease of the intensity of the solar spectrum toward short wave-length at the base of our atmosphere:

TABLE X

λ	Outside the Atmosphere			Log. of Total Opacity of Atmosphere June 7	At the Surface of the Earth	
	Log. of Photo. Int.	Intensity in Energy			Intensity in Energy	
		Log I_0	$I_0\,10^{-4}$		Log I_1	I_1
3143	5.78	5.19	15.5	0.84	4.35	22400
3104	5.73	5.17	14.7	0.99	4.18	15100
3052	5.85	5.41	25.6	1.40	4.01	10200
3022	5.57	5.20	15.8	1.77	3.43	2700
2997	5.64	5.34	21.8	2.22	3.12	1320
2963	5.43	5.22	16.6	3.10	2.12	132
2956	5.41	5.21	16.2	3.33	1.88	76
2946	5.31	5.13	13.5	3.73	1.40	25
2936	5.31	5.15	14.1	4.12	1.03	11
2931	5.25	5.10	12.6	4.36	0.74	5.5
2922	5.30	5.16	14.5	4.82	0.34	2.2
2917	5.16	5.02	10.5	5.08	9.94	0.87
2912	5.00	4.87	7.4	5.39	9.48	0.30
2906	4.52	4.40	2.5	5.78	8.62	0.04
2898	4.76	4.65	4.5	6.36	8.29	0.02

between λ 3143 and λ 2898 the ratio of the intensities is about a million for the sun at the zenith. This great fall in intensity is not due to a diminution in the emission of the sun, but to the almost complete opacity for the short wave-length of our atmosphere, which does not permit the passage of more than one part in two millions of the radiation at λ 2898.

We see that aside from certain irregularities due to the presence of very many lines of solar origin, there is no evidence of any low point in the intensity in the region the best studied from λ 3143 to λ 2922. Beyond, from λ 2917 to λ 2898, the measures are less sure and they only show that it is not a very rapid falling off. Hence it is certain that the limitation of the spectrum does not

exist outside our atmosphere and that it is not due to an absorption having its origin in the solar atmosphere.

A comparison of the intensities of the radiations emitted by the center and the edge of the solar disk tends to confirm this conclusion. We have established the fact that the spectrum of the limb is about two times less intense than that of the center, without any appreciable difference from one end to the other of the spectral region studied; there should be a weakening more and more marked toward short wave-lengths, if the strata of the solar atmosphere were responsible for the limitation of the spectrum.

10. *Daily variation in the quantity of ozone.*—The method explained above gives the quantity of ozone in absolute units. It has the inconvenience of requiring a long series of exposures

TABLE XI

	λ 3143	λ 3104	λ 3052	λ 3022	λ 2997	λ 2956	λ 2946	λ 2936	λ 2931	λ 2922
Log I June 10	5.30	5.13	4.72	4.04	3.55	1.93	1.51	0.97	0.74	0.30
Log I June 21	5.35	5.17	4.84	4.15	3.80	2.24	1.94	1.54	1.31	0.84
Difference	0.05	0.04	0.12	0.11	0.25	0.31	0.43	0.57	0.57	0.54

in the course of a day, where the measures and computations are complicated. A method of comparison permits the study of the variation of the quantity of ozone by a single exposure made at noon each day.

An exposure is made under identical conditions on one and the same plate each day; once we also make a further exposure through a filter in order to have the necessary data for calibrating the plate. We thus obtain on a single plate the data relative to a score of different days. Thus we may deduce the values of the photographic intensities of the different rays for corresponding days. If the quantity of ozone should remain constant from one day to another, all the radiations would be modified in the same ratio by the effect of a change in the haziness. It is not so, however, for the distribution of intensities in the spectrum undergoes great changes from one day to the other. We give as an example those of June 10 and 21, 1920. Table XI gives for noon of these two days the logarithms of the photographic intensities of the different rays.

We see that the intensities are almost the same for λ 3143 and that the differences increase regularly so that at λ 2922 the spectrum is nearly three times weaker on June 10 than on June 21. The thickness of ozone traversed is thus notably greater on the tenth than on the twenty-first; as the zenith distance is almost the same, it is necessary to conclude that a greater quantity of ozone was present in the atmosphere.

We may further calculate for each day the quantity of ozone. Let I and I' be the intensities of the two radiations measured on the plate. We know further their intensities outside the atmosphere, I_0 and I_0, as given in Table X. The coefficients of absorption of the ozone are α and α' and those of the weakening by molecular diffusion are β and β'; finally the haze produces the same weakening, δ, on the two rays. We may write the two relations:

$$\log I = \log I_0 - (\alpha x + \beta) \sec z - \delta$$

and

$$\log I' = \log I'_0 - (\alpha x + \beta') \sec z - \delta .$$

In order to eliminate δ we subtract member from member, whence we obtain:

$$x = \frac{(\log I' - \log I) - \log I'_0 - \log I_0) - (\beta - \beta') \sec z}{(\alpha - \alpha') \sec z} .$$

We may thus combine any two rays whatever, but it is evidently desirable to choose two separate radiations among those measured under good conditions. For λ 2931 and λ 3104 the preceding formula becomes:

$$x = \frac{\log I' - \log I - 0.64}{10.1 \sec z} - 0.01 .$$

Table XII gives for some of the days of May and June, 1920, values of x, the thickness of pure ozone, reckoned along the vertical in cm and at normal pressure:

These variations are evident. The other determinations made in the summer of 1919 led to the same result with variations of the same order. These changes appear irregular and of short period; in so short an interval as we have been able to follow them, we have not been able to notice any relation between them and the atmospheric conditions. It would be interesting to observe them

in a continuous manner, a task which would not present any difficulty but could be undertaken only by an institution having a sufficiently large personnel.

11. *Localization and origin of the atmospheric ozone.*—We now know the quantity of ozone to be found in our atmosphere, but we do not know where it is situated. If distributed uniformly, its proportion by volume would be 4×10^{-7} or 0.4 cubic cm per cubic meter. This is much more than the amount found in the

TABLE XII

May 1920	Thickness cm of Ozone	June 1920	Thickness cm of Ozone
21..........	0.304	4..........	0.293
25..........	0.310	5..........	0.297
27..........	0.298	7..........	0.325
28..........	0.290	9..........	0.321
29..........	0.275	10..........	0.335
31..........	0.306	11..........	0.314
		21..........	0.286
		23..........	0.289

air near the surface of the earth and in the regions of the atmosphere which we can reach. The best determinations by the chemists give a very much smaller quantity. When we gave a brief summary of our first results in 1913[1] we suggested a method for studying the presence of ozone in the lower atmosphere. In 1918 R. J. Strutt[2] made the experiment, which consisted in studying the absorption of ultra-violet radiation of an artificial source between two stations separated by several kilometers. Strutt found that the mercury ray λ 2536 is transmitted to an appreciable extent through 6.45 km of air. If in that distance the air contained the proportion of ozone given above (0.4 cubic cm per cubic meter) the intensity of the ray would be reduced in proportion of $10^{33}:1$, so that it would have been absolutely unobservable. Hence it is perfectly certain that ozone is not distributed uniformly in the atmosphere.

It is furthermore highly improbable that the necessary quantity of ozone is situated in the stratum of air accessible to man. Observations of the solar spectrum made at very high altitudes have

[1] *Journal de Physique* (5), **3**, 196, 1913.

[2] *Proceedings of the Royal Society*, A, **94**, 260, 1918.

shown that its limits undergo only very small variations with altitude. Therefore it is necessary for us to locate the great part of the ozone in the most elevated strata, possibly above 50 km. It is easy to explain its presence in this region, because we know that oxygen is transformed into ozone by radiation from the very short waves, shorter than λ 2000. The presence of these rays in the radiation emitted by the sun is very probable, because we do not see any limitation of the spectrum in the region we have studied. These rays should therefore ozonize the atmosphere, but being strongly absorbed by oxygen, are not able to penetrate very far and act only upon the very first strata. The ray λ 1800 is completely absorbed by a dozen meters of air at ordinary pressure; it is therefore incapable of penetrating at an altitude of less than 40 km. Hence it is not astonishing that the ozone should not be formed in the strata accessible to us. Furthermore, the rays between λ 2000 and λ 3000 decompose ozone. A state of equilibrium is established between these opposing actions, and the quantity of ozone which exists depends upon the relative proportion of the intensities of these two spectral regions. The variations observed in the quantity of ozone may have as their principal cause the variations in the solar radiation and in particular the region of exceedingly short rays, which unfortunately is not observable.

It would be interesting to follow the variations of the ozone through the solar period and further, in a set of daily observations, to compare them with measures of the solar constant which has been found by Abbot to vary. If an extensive collaboration was possible, as has been undertaken for other phenomena, it would be interesting to follow these observations at different points on the surface of the earth.

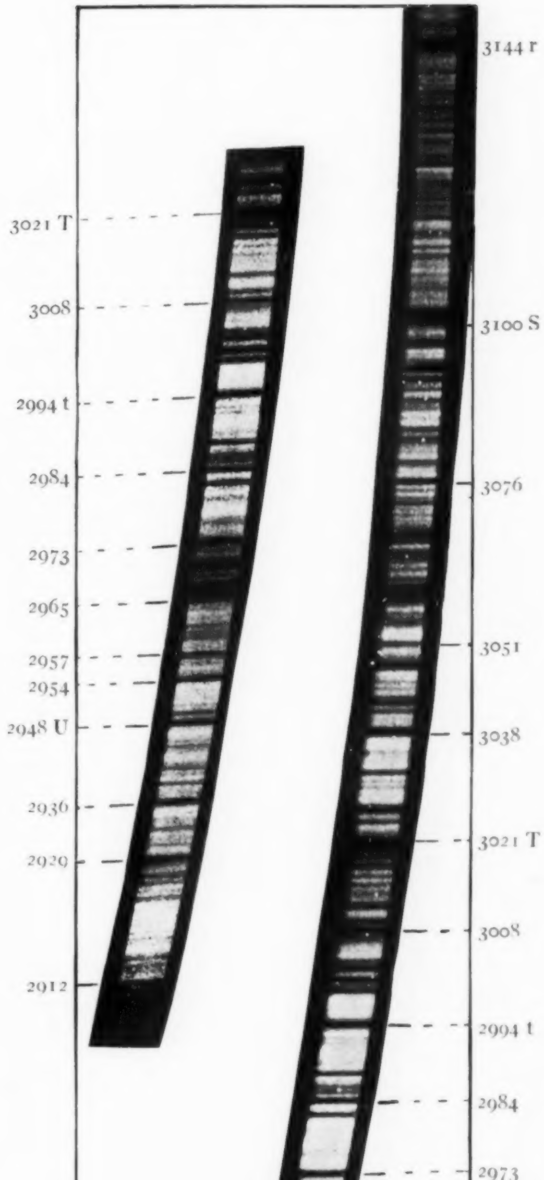
We might be able to obtain some idea as to the altitude of the stratum of the ozone by the following method. In consequence of the curvature of the earth, the law for sec z, which gives the thickness traversed, is no longer exact when the zenith distance is large. The complete law involves the altitude of the absorbing strata. By comparing the theoretical law with the results of observation, it might be possible to get an idea of the altitude at which absorption takes place. It would be necessary for that to make measures almost to the horizon; here it would not be possible

to use the region at λ 3000, which is totally suppressed before the sun has reached the horizon. We might employ the weak bands of ozone in the region λ 3300, the presence of which has been brought out by Fowler and Strutt in the spectrum of the stars and of the sun when near the horizon.

12. *Photographic map of the extremity of the solar spectrum.*—It seemed to us to be desirable to obtain a photographic map of the extremity of the solar spectrum. We do not know enough about this. Rowland's map stops at λ 3075, but that region is much disturbed by the superposition of the spectrum of another order. The map by Higgs, which is better in that region, does not go beyond λ 3000. Cornu published a drawing which extended to λ 2920, based on the photographs made by Simony at the Pic of Teneriffe, but such a drawing does not have the same authentic value as a direct reproduction. We have succeeded in obtaining a plate which has in all that part of the spectrum a suitable exposure in spite of the great variation in the intensity, by varying the exposure time along the region of the spectrum. For this purpose a shutter, movable by hand by a micrometer screw, was displaced in the focal plane of the first spectroscope in accordance with a law experimentally determined for making the impression almost uniform through the whole spectrum. This shutter was moved toward short wave-lengths, so as to occult successively the different regions of the spectrum; in this way, in going from λ 3020 to λ 2930, the time of exposure varied in the ratio of from 1:1000. We made thus two plates, one containing the region from wave-lengths shorter than λ 3022, the other extending from λ 3000 to λ 3150. Plate III gives a positive reproduction of these two spectra, enlarged about 10 times. The original plates have been measured, and with the use of the iron lines in the solar spectrum as standards, we have obtained a table of wave-lengths between λ 2898 and λ 3050. This table will soon be published in the *Journal de Physique*.

University of Marseille
Faculty of Sciences
December 1920

PLATE III



MAP OF ULTRA-VIOLET END OF SOLAR SPECTRUM

STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS

NINETEENTH PAPER: A PHOTOMETRIC SURVEY OF THE PLEIADES[1]

By HARLOW SHAPLEY AND MYRTLE L. RICHMOND

ABSTRACT

Survey of photographic and photo-visual magnitudes of 821 stars in four square degrees in the Pleiades.—The photometric study of a region two degrees square, with Alcyone in the center, was undertaken to supplement Trumpler's study of the proper motions. The results of measurement of 30 photographs made with the 60-inch reflector, comprising a total of 290 exposures, are given in Table II. There are no abnormally large color-indices. This is the first star cluster for which *comprehensive data for dwarf stars* have been secured.

A detailed study of the proper motions of over a thousand stars in the neighborhood of the Pleiades by Dr. Trumpler has shown that the cluster contains in the central field of 4 square degrees about 200 members brighter than the fourteenth magnitude. The parallax of the Pleiades is of the order of 0.″01; hence the absolute magnitudes of the main bulk of the cluster stars lie between +5 and +9. As this is one of the few stellar groups in which the color, frequency, distribution, and motions of dwarf stars can be investigated successfully, a study of the photographic and photovisual magnitudes was begun at Mount Wilson three years ago in order to supplement Dr. Trumpler's work on proper motion.

The comparison of the Mount Wilson magnitudes and colors with the work of other observers will be made in Trumpler's monograph, which will also include a full discussion of the structure of the system of the Pleiades.

For each catalogued magnitude the average number of images measured is approximately four, and the average probable error is of the order of 0.06 mag. Only the final results of the photometric work are given in the present paper. Information relating

[1] *Contributions from the Mount Wilson Observatory*, No. 218.

to the photographs, measures, and reduction for any individual star can be obtained by communicating with the Mount Wilson Observatory.

In order to cover with the 60-inch reflector a region 2 degrees square (Alcyone at the center), it was necessary to divide the area

TABLE I

PHOTOGRAPHS OF THE PLEIADES

Plate Number	Kind	Date	Fields
4072	S 27	1917 Oct. 12	19, 20, 21, 22, 23, 24
4073	S 27	Oct. 12	25, 26, 27, 28, 29, 30
4183	Iso	Nov. 11	20, 23, N. P.
4184	S 27	Nov. 11	20, 23, N. P.
4185	S 27	Nov. 11	23, 24, N. P.
4300	Iso	Dec. 20	13, 14, 15, 16, 17, 18
4301	S 27	Dec. 20	13, 14, 15, 16, 17, 18
4680	S 27	1918 Sept. 1	31, 32, 33, 34, 35, 36
4681	Iso	Sept. 1	31, 32, 33, 34, 35, 36
4682	S 27	Sept. 1	1, 7, 13, 19, 25, 31
4765	Iso	Nov. 1	12, 10, N. P.
4766	S 27	Nov. 1	12, 10, N. P.
4767	S 27	Nov. 1	10, 7, N. P.
4768	Iso	Nov. 1	7, 8, 9, 10, 11, 12
4769	S 27	Nov. 1	7, 8, 9, 10, 11, 12
4849	Iso	1919 Feb. 25	19, 20, 21, 22, 23, 24
4850	S 27	Feb. 25	1, 2, 3, 4, 5, 6, N. P.
4851	Iso	Feb. 25	6, 3, N. P.
4852	S 27	Feb. 25	6, 3, N. P.
5026	S 30	July 23	6, 12, 18, 24, 30, 36, N. P.
5027	S 30	July 23	36, 33, N. P.
5043	Iso	Aug. 22	25, 26, 27, 28, 29, 30
5056	Iso	Sept. 21	1, 2, 3, 4, 5, 6
5118	Iso	Oct. 31	6, 12, 18, 24, 30, 36
5154	S 30	Dec. 18	15, 16, 22, N. P.
5155	S 30	Dec. 18	21, 22, 27, 28, N. P.
5607	Iso	1920 Sept. 14	28, 22, 21, N. P.
5608	S 30	Sept. 14	22, 21, N. P.
5609	S 30	Sept. 14	21, 15, 16, 22
5610	Iso	Sept. 14	22, 16, 8, 2, 20, 26

into 36 equal fields, the centers of adjacent fields being separated by about 20′.

The scales of the photographic and photovisual magnitudes were determined by comparison of these fields with the North Polar standards[1] and with each other. The list of photographs in Table I shows how the various fields were intercompared and connected with the Pole. Each plate was given two separate exposures

[1] Seares, *Mt. Wilson Contr.*, No. 97; *Astrophysical Journal*, **41**, 206, 1915.

to each of three or more fields, with exposure times of one minute and five minutes, respectively, for photographic and photovisual magnitudes. Yellow filter C was used with the isochromatic plates (Cramer Instantaneous). The measures and reductions were made according to the methods generally used for photometry at Mount Wilson.

The photometric catalogue, Table II, contains in the first three columns the number in Trumpler's forthcoming catalogue and the abbreviated positions for 1900.0 from the same source. The fourth and fifth columns contain the photographic magnitude and color index determined at Mount Wilson. In the sixth column the first number for each star refers to photographic plates, the second to photovisual plates.

All stars brighter than 11.0 photographic, or 10.0 photovisual, are omitted from our catalogue. Those stars whose magnitudes are disqualified by the proximity of the photographic images of other stars are also omitted. A few of the stars catalogued are probably variable; the data relative to them will be published by Trumpler.

The color indices of 753 stars are given in Table II; five of them are negative; only two exceed +2.00. For 26 stars there are photographic magnitudes, but no color indices; for 42 stars there are photovisual magnitudes only. The total number of stars in our catalogue is 821.

TABLE II

Catalogue of 821 Stars in the Pleiades

Trumpler No.	R.A. 1900	Decl. 1900	Pg. Mag.	C.I.	No. of Plates	Trumpler No.	R.A. 1900	Decl. 1900	Pg. Mag.	C.I.	No. of Plates
	3^h37^m						3^h37^m				
1	$6^s.3$	24° 41′.3	14.54	0.35	3,3	68	$58^s.9$	23° 56′.0	11.09	0.99	1,1
3	7.9	24 29.3	11.89	......	1,.	69	59.3	23 26.1	13.24	0.61	3,4
4	8.5	24 47.7	13.17	1.61	1,1	71	59.8	24 12.9	......	......	.,1
5	11.0	24 33.0	14.00	0.49	4,1		3^h38^m				
9	12.6	23 1.4	14.14	0.63	2,1	72	2.0	23 55.8	13.85	0.41	1,1
10	16.0	22 50.4	13.61	1.06	2,1	73	2.3	24 8.3	12.29	0.27	1,2
11	16.8	23 36.8	14.30	0.72	2,2	76	3.2	23 49.8	11.62	1.27	2,3
12	16.9	23 17.2	......	......	1,1	77	4.1	23 53.1	11.79	0.47	1,1
13	17.1	23 51.4	13.92	1.07	1,1	78	5.2	22 54.6	13.86	0.81	2,1
14	17.8	24 46.5	13.73	0.73	3,3	79	5.6	23 13.6	11.32	−0.08	1,2
16	20.1	23 16.0	13.04	1.94	1,1	82	7.0	23 48.9	14.18	0.54	3,3
17	20.3	24 29.6	14.72	0.47	4,1	83	7.0	23 55.6	13.07	0.31	1,1
18	21.0	23 14.7	13.67	0.59	1,1	84	7.1	23 17.0	13.07	0.74	1,2
19	22.7	23 56.1	14.43	1.71	1,1	85	7.5	23 52.3	13.07	−0.09	1,1
20	22.8	24 5.8	12.52	0.38	1,1	86	8.1	24 25.9	13.75	0.67	1,1
21	23.0	22 55.7	13.24	1.53	2,1	87	8.1	24 41.2	13.97	0.69	3,3
22	23.2	24 39.9	14.39	0.85	3,3	88	8.1	23 45.4	14.34	0.47	3,2
23	23.7	23 32.5	14.29	0.54	3,3	89	8.2	23 57.6	12.08	0.32	1,1
24	24.2	24 22.2	14.55	1.15	1,1	90	8.5	24 4.8	14.09	1.37	1,1
25	25.3	23 16.7	14.09	0.39	1,2	91	9.2	24 1.6	13.95	0.51	1,1
26	25.5	24 10.1	......	......	.,1	92	9.7	24 1.2	14.03	0.59	1,1
27	25.7	23 42.8	13.94	0.54	2,2	93	10.4	24 12.2	......	......	.,1
28	26.5	24 31.1	14.27	1.05	4,3	94	13.4	23 55.8	......	......	.,1
29	27.5	22 56.8	13.68	0.73	2,1	95	15.0	24 25.2	11.65	0.41	1,1
31	28.7	24 18.0	14.03	1.01	4,4	96	15.2	24 18.4	14.63	......	1,.
32	28.9	22 58.3	12.12	1.62	2,1	97	15.8	24 14.4	......	......	.,1
33	29.7	23 21.2	......	......	.,1	98	16.1	24 27.8	......	......	2,2
34	30.4	22 54.5	......	......	2,1	99	16.2	24 5.2	11.20	0.70	1,1
35	34.2	24 38.6	14.42	0.64	4,3	100	16.6	23 3.8	12.15	0.82	2,2
36	34.3	24 20.1	12.23	0.47	1,1	101	16.6	23 43.0	12.64	0.62	2,2
37	36.8	23 21.4	11.04	0.34	1,2	102	16.6	24 45.4	12.33	1.09	3,3
38	37.4	24 27.8	14.90	1.37	4,4	103	17.0	23 42.9	12.28	0.58	2,2
39	37.5	24 16.5	14.49	0.62	1,1	106	20.0	23 57.4	......	......	1,1
40	37.7	24 16.2	14.52	0.80	1,1	107	21.5	22 51.3	12.91	0.58	2,1
41	39.0	23 26.7	11.94	0.65	3,3	108	22.2	24 28.8	11.04	0.31	1,2
42	39.1	24 36.1	12.66	1.26	4,3	111	22.5	22 49.2	13.80	0.92	2,1
43	40.6	22 52.3	14.63	1.68	1,1	112	22.5	24 14.5	......	......	.,1
44	42.0	24 25.4	14.03	0.23	1,1	113	22.7	24 27.4	11.55	0.93	2,2
45	42.1	24 32.4	12.58	0.43	4,3	114	22.9	23 56.7	12.26	0.06	1,1
46	42.9	23 13.1	......	......	.,2	115	24.1	24 27.2	14.14	0.87	2,2
48	44.9	22 56.8	13.15	0.39	2,1	117	25.6	23 12.1	13.88	1.07	1,1
49	45.1	23 28.3	14.40	0.50	3,3	119	27.4	24 6.6	13.53	1.39	1,1
52	46.9	24 45.7	14.18	1.80	3,3	120	28.4	23 21.1	12.13	0.85	1,1
54	49.0	24 39.0	12.18	0.87	4,3	122	28.9	24 16.4	......	......	1,2
55	49.5	24 38.4	13.43	0.36	4,3	123	32.0	24 23.9	13.85	0.37	1,1
56	50.0	24 41.2	12.34	0.67	3,3	124	34.8	23 3.8	14.09	0.17	1,1
57	52.0	23 43.3	12.16	0.72	2,2	124a	34.9	23 3.9	14.89	0.49	1,1
58	53.3	22 53.7	11.42	1.26	2,1	125	38.4	23 22.4	13.71	0.84	1,2
59	53.6	24 44.7	14.14	0.45	3,3	126	38.5	24 43.4	14.21	0.49	5,5
60	54.2	23 55.8	11.76	0.36	1,1	127	39.0	24 38.8	14.41	0.71	5,5
61	56.5	23 14.6	15.12	1.14	1,2	128	39.1	23 24.4	13.64	0.57	1,1
62	57.0	22 56.3	13.61	1.56	2,1	129	39.1	23 44.8	13.22	1.80	2,2
63	57.2	23 2.8	11.70	0.87	2,1	129a	39.3	23 41.2	15.68	1.48	2,1
66	58.2	24 44.4	14.71	0.74	3,3	130	42.2	24 11.7	11.89	0.76	1,1
67	58.3	23 22.8	13.46	0.54	1,2						

TABLE II—*Continued*

Trumpler No.	R.A. 1900	Decl. 1900	Pg. Mag.	C.I.	No. of Plates
131....	3^h38^m 42.8	24°30'.2	......	......	.,1
132....	44.7	24 5.0	12.42	0.62	2,2
133....	44.9	23 2.9	14.02	0.62	2,1
134....	45.2	23 58.2	13.81	0.55	4,3
135....	45.8	24 35.3	14.64	0.49	5,3
136....	46.2	24 44.6	13.90	0.58	4,3
137....	47.3	23 55.0	12.11	0.64	4,3
138....	48.6	24 24.9	13.41	1.29	2,1
140....	51.9	24 35.8	14.50	1.14	4,3
141....	52.6	24 34.4	14.12	0.62	3,3
142....	52.7	23 50.8	13.24	1.78	3,2
143....	52.9	23 44.3	13.28	......	1,.
144....	53.2	24 6.1	14.95	0.84	3,2
145....	53.3	23 16.4	11.13	......	1,.
146....	55.0	23 4.5	14.11	0.75	2,1
147....	55.7	24 37.7	14.20	0.63	3,3
150....	57.0	24 24.9	14.03	0.91	1,1
151....	57.9	23 36.4	11.62	0.71	2,1
152....	58.7	24 28.4	13.04	1.12	1,3
153....	59.3	24 7.1	11.02	0.60	3,3
154....	59.9	23 3.2	13.95	0.64	3,1
154a....	3^h39^m 0.7	23 46.5	14.90	1.33	4,2
155....	2.4	23 38.0	14.84	1.11	1,1
156....	3.7	24 16.2	13.52	1.80	1,1
157....	5.3	23 46.6	14.12	0.56	4,2
157a....	6.9	23 9.4	14.94	1.26	1,2
158....	7.5	23 31.5	13.11	1.30	1,1
159....	8.4	24 40.3	14.61	0.79	2,2
160....	8.5	23 4.8	13.98	0.53	4,2
161....	9.0	24 43.0	12.82	0.40	2,2
164....	10.4	24 27.4	14.00	0.36	2,3
165....	10.9	24 21.6	13.98	0.54	1,1
167....	12.7	24 10.2	11.56	0.43	2,1
167a....	12.8	24 28.9	15.60	1.26	3,2
168....	14.8	24 46.4	14.49	0.74	2,2
169....	15.1	23 34.8	13.44	0.92	1,1
170a....	16.0	24 30.7	15.74	1.16	2,2
171....	16.1	23 48.6	12.86	1.26	3,2
172....	16.9	24 43.1	14.36	0.62	2,2
173....	17.1	24 37.2	13.25	0.62	2,2
174....	18.2	22 58.9	13.03	0.85	3,1
175....	18.4	24 9.7	15.00	1.42	2,3
176....	19.3	24 34.4	12.45	0.38	2,2
178....	19.8	24 24.6	12.20	0.36	1,1
179....	22.0	24 33.4	14.02	0.49	2,2
181....	22.6	23 50.3	13.68	0.49	3,2
182....	22.8	24 40.5	14.64	0.69	2,2
183....	23.1	23 51.9	11.23	0.39	3,2
184....	23.5	24 18.4	14.60	0.65	1,1
185....	24.1	23 47.6	14.42	0.71	4,2
186....	25.6	23 24.8	13.22	0.70	2,3
187....	3^h39^m 25.9	24°34'.3	......	......	2,2
188....	26.1	24 25.1	14.81	0.84	1,1
189....	27.2	24 24.8	14.66	1.00	1,1
190....	27.2	23 9.5	13.15	0.67	1,2
192....	29.5	23 45.3	14.05	0.59	4,2
193....	29.5	23 10.3	12.58	0.67	1,2
194....	29.7	24 12.6	14.32	0.79	1,1
196....	30.4	24 18.0	14.12	0.72	1,1
197....	31.1	23 7.0	12.90	0.82	4,3
199....	32.5	23 55.8	13.80	1.46	3,2
200....	33.0	22 48.0	13.12	0.94	3,1
201....	33.0	22 56.6	13.82	0.98	3,1
202....	33.2	23 5.1	13.00	0.64	4,3
203....	33.9	23 26.8	14.80	1.22	2,3
204....	35.1	24 42.2	14.09	0.69	2,2
205....	35.8	23 33.4	13.98	1.06	1,1
206....	36.0	23 59.4	14.02	0.96	3,2
207....	36.0	23 18.4	13.83	0.80	1,2
207a....	37.0	23 41.5	15.98	1.38	1,1
209....	38.1	22 57.2	13.98	0.58	3,1
210....	38.4	24 35.3	11.84	1.07	2,2
211....	38.8	24 11.2	12.54	0.48	1,1
212....	38.8	23 36.8	13.92	0.64	1,1
213....	39.0	23 46.2	......	......	4,2
214....	39.8	23 2.5	14.18	0.78	3,1
216....	41.2	23 36.0	13.98	0.83	1,1
217....	42.5	24 18.8	......	......	1,1
218....	43.7	24 36.3	14.16	0.63	2,1
220....	43.7	23 26.5	13.38	1.18	2,3
221....	43.8	24 35.6	......	......	2,2
222....	43.8	24 44.6	14.32	1.07	2,2
223....	44.5	23 58.5	......	......	2,2
224....	44.6	24 7.1	12.12	0.82	4,3
225....	46.5	23 57.4	12.64	0.68	3,2
226....	46.7	24 24.0	12.48	1.45	1,1
227....	47.1	24 40.4	12.84	0.86	3,2
228....	47.7	23 54.4	11.46	0.48	3,2
228a....	47.7	23 45.7	15.20	0.69	2,1
229....	50.3	24 33.9	13.59	0.64	2,2
230....	51.2	23 11.2	......	......	1,1
232....	52.5	24 6.8	12.12	1.18	4,2
236....	55.3	23 18.4	13.44	1.14	1,2
237....	56.2	24 41.2	14.94	0.64	3,3
238....	56.3	24 29.3	12.93	0.68	1,2
239....	56.7	23 7.1	12.44	0.24	1,2
241....	57.8	23 56.3	13.75	1.84	3,2
241a....	59.9	23 8.6	14.70	1.20	1,2
242....	3^h40^m 0.1	22 49.8	13.85	1.01	4,1
243....	0.5	24 3.8	14.11	0.61	3,2
244....	0.7	24 44.3	......	......	2,2
244a....	1.8	24 12.8	15.04	0.80	1,1
245....	4.0	24 31.6	12.84	0.50	3,2
246....	4.1	24 6.6	14.29	0.66	1,1
248....	7.7	23 43.9	12.52	......	2,.
249....	7.1	23 25.5	......	......	.,2
250....	7.3	24 40.8	11.10	0.64	2,3
251....	7.6	24 42.3	11.02	0.12	3,3

TABLE II—*Continued*

Trumpler No.	R.A. 1900	Decl. 1900	Pg. Mag.	C.I.	No. of Plates	Trumpler No.	R.A. 1900	Decl. 1900	Pg. Mag.	C.I.	No. of Plates
	3^h40^m						3^h40^m				
252....	$7^s.9$	24°26'.0	14.40	1.25	2,1	317....	$40^s.0$	22°48'.1	14.35	0.44	2,1
253....	8.9	24 15.3	14.03	1.41	1,1	318....	40.1	24 1.9	12.89	0.71	2,2
254....	9.0	24 34.2	14.30	0.89	3,3	319....	40.1	24 38.9	11.12	0.69	1,2
255....	9.1	24 31.7	13.96	0.78	2,2	321....	40.4	24 37.7	14.20	1.08	2,2
257....	9.3	24 15.0	14.66	1.19	1,1	322....	41.3	24 12.6	14.52	0.96	1,1
258....	9.4	22 59.8	14.11	0.99	4,1	323....	41.7	24 30.5	13.76	0.61	2,3
259....	9.4	24 33.3	12.88	0.74	2,3	324....	41.8	23 28.6	....	......	2,2
260....	9.9	22 47.8	14.08	1.04	2,1	325....	42.0	24 35.4	14.04	0.51	2,2
261....	10.9	24 36.3	13.98	1.12	3,3	326....	42.3	23 41.0	13.58	0.95	2,1
262....	10.9	24 37.1	13.85	0.53	3,3	328....	42.7	24 2.4	13.77	0.26	2,2
263....	11.4	24 35.8	14.46	1.08	3,3	329....	42.9	23 43.1	12.80	0.43	2,1
264....	11.4	24 19.3	14.07	0.73	1,2	330....	45.0	23 11.2	12.83	0.52	1,1
265....	11.6	23 44.9	11.28	......	2,.	332....	46.5	24 37.1	13.75	0.63	2,2
266....	11.6	22 54.4	11.25	0.88	4,1	333....	46.6	23 56.7	14.20	0.79	2,2
267....	12.9	22 49.2	13.20	1.59	2,1	333a....	46.7	23 56.7	15.29	1.07	2,2
268....	14.0	24 18.6	12.51	0.70	2,2	334....	47.2	24 0.1	10.92	0.28	2,2
269....	15.0	22 59.4	14.64	1.17	2,1	335....	47.3	23 1.8	14.06	0.30	2,1
271....	15.7	22 49.4	13.96	......	2,.	336....	47.5	24 19.0	14.17	0.18	1,1
272....	16.3	24 43.8	13.74	0.70	3,3	339....	49.0	24 17.5	14.47	0.74	1,1
274....	17.2	23 53.3	13.85	0.65	2,4	340....	49.3	24 35.3	14.28	0.55	2,2
275....	17.5	24 18.7	14.75	1.28	2,1	341....	49.6	23 30.9	13.47	0.32	2,1
276....	17.7	24 45.0	14.19	0.63	3,2	342....	50.5	24 14.9	13.41	0.19	1,1
277....	17.8	22 53.8	14.03	0.88	3,1	343....	52.3	22 50.5	12.48	0.89	2,1
278....	18.4	22 49.0	....	......	.,1	344....	54.5	24 39.2	13.85	0.41	1,2
280....	19.2	23 40.4	13.79	0.61	2,1	344a....	55.2	24 13.3	14.84	0.42	1,1
282....	20.4	24 33.5	12.30	0.66	1,1	345....	55.6	23 51.7	14.43	0.53	2,2
283....	20.7	24 30.6	13.35	0.39	1,1	347....	56.6	24 36.6	11.90	0.71	2,2
283a....	21.0	23 36.4	15.35	......	3,.	348....	56.6	24 29.2	....	......	.,3
283b....	21.9	24 1.2	14.55	0.73	2,2	349....	57.7	22 53.9	11.52	0.99	2,1
285....	22.8	23 10.5	11.55	0.65	1,1	350....	58.3	23 16.4	11.00	0.22	1,1
							3^h41^m				
287....	23.5	23 43.4	13.58	0.30	2,1	350a....	0.3	24 10.5	15.18	0.88	3,3
287a....	23.5	23 36.3	14.87	0.51	2,1	351....	0.5	23 30.4	13.66	0.51	2,1
288....	24.5	23 12.5	13.17	0.76	1,2	352....	1.3	24 46.6	13.45	1.29	1,1
289....	25.0	24 15.5	11.00	0.36	1,1	353....	1.6	22 59.8	13.98	0.76	2,1
290....	$25^s.1$	24 38.4	14.05	0.37	2,2	355....	2.4	23 46.7	12.60	0.34	4,3
291....	27.0	24 3.0	13.52	0.71	2,2	356....	2.4	24 4.4	11.30	0.40	2,2
292....	27.2	24 42.3	11.80	1.62	2,2	357....	2.6	24 1.2	12.68	1.36	2,2
293....	27.5	24 38.1	13.58	0.40	2,2	358....	4.0	23 40.3	12.69	0.85	2,1
294....	29.5	23 0.6	11.62	1.20	2,1	359a....	5.5	23 33.8	14.41	0.02	2,1
296....	31.0	24 7.4	11.51	0.33	1,2	360....	5.8	24 19.5	12.29	1.90	1,1
297....	32.4	23 16.9	13.56	0.90	1,2	361....	6.1	23 11.1	12.39	0.96	1,1
298....	32.8	23 3.8	14.15	0.59	2,1	362....	6.7	22 57.5	14.12	0.70	2,1
299....	33.3	24 44.8	13.68	0.70	2,2	363....	7.0	22 49.6	13.86	0.58	2,1
300....	33.7	23 4.0	14.34	0.68	2,1	364....	7.1	23 23.8	14.12	0.32	1,1
302....	34.4	22 47.6	14.06	0.54	2,1	366....	10.2	24 33.6	12.67	0.39	1,2
303....	34.5	24 1.3	13.74	0.26	2,2	367....	10.4	23 7.4	13.92	0.44	2,2
304....	34.7	24 0.7	13.93	0.27	2,2	368....	10.6	24 12.9	14.01	1.07	1,1
305....	34.9	24 36.9	14.38	0.06	2,2	369....	11.0	23 24.3	12.87	0.56	1,1
305a....	35.4	23 58.2	15.36	0.90	2,2	370....	11.9	23 10.2	13.79	0.41	1,1
306....	35.6	22 52.3	12.32	0.55	2,1	370a....	12.8	23 24.0	14.32	1.10	1,1
306a....	35.6	23 40.3	14.59	1.03	2,1	371....	12.9	24 46.3	14.79	0.77	1,1
307....	36.1	23 33.0	11.41	0.08	2,1	373....	13.2	24 48.4	11.52	0.30	1,2
308....	36.4	24 16.6	14.21	0.41	1,1	374....	13.3	24 7.5	14.07	0.56	3,3
309....	37.4	23 49.6	12.23	0.32	3,2	375a....	14.3	23 44.0	14.59	0.34	2,2
310....	37.5	23 32.1	13.41	1.07	2,1	376....	14.3	23 53.6	14.16	0.34	2,2
312....	38.7	24 27.5	13.86	0.10	1,2	377....	16.0	24 29.4	14.75	0.77	2,1
313....	38.8	23 39.0	11.90	−0.06	2,1	378....	17.7	23 24.2	13.17	0.68	1,1
314....	38.8	23 29.3	13.78	0.58	2,2	379....	18.0	23 48.4	13.86	0.48	4,3
315....	38.8	23 40.7	13.37	0.62	2,1	380....	18.3	23 53.0	13.03	0.59	2,2
316....	39.7	24 26.2	12.70	0.52	1,1	382....	20.4	23 30.5	11.50	0.83	2,1

TABLE II—*Continued*

Trumpler No.	R.A. 1900	Decl. 1900	Pg. Mag.	C.I.	No. of Plates	Trumpler No.	R.A. 1900	Decl. 1900	Pg Mag.	C.I.	No. of Plates
	$3^{\mathrm{h}}41^{\mathrm{m}}$						$3^{\mathrm{h}}41^{\mathrm{m}}$				
383....	$20^{\mathrm{m}}.7$	24° 4'.8	13.62	0.86	2,2	451....	$57^{\mathrm{m}}.0$	23°29'.0	13.85	0.50	3,2
384....	20.9	23 36.2	12.44	1.42	3,2	452....	58.0	23 30.8	13.80	0.78	2,2
385....	21.2	24 8.6	12.96	1.06	3,3	453....	58.3	23 59.6	13.96	0.42	4,5
385*a*....	21.2	23 47.4	15.17	0.83	3,3	454....	58.8	24 42.2	11.77	1.01	4,3
386....	21.4	24 19.9	13.93	0.25	3,2	455....	59.0	22 54.2	14.26	0.40	3,2
388....	21.8	22 56.5	13.99	0.71	4,1	456....	59.0	24 27.2	13.34	0.74	4,3
389....	22.4	24 33.8	13.39	0.68	4,2	457....	59.2	23 22.8	13.72	1.05	2,3
390*a*....	22.6	23 38.4	14.66	0.62	3,1	458....	59.8	23 7.7	....	......	.,4
392....	23.2	23 41.5	13.92	0.72	3,2						
393....	23.5	24 39.3	14.82	0.75	5,3						
							$3^{\mathrm{h}}42^{\mathrm{m}}$				
394....	24.1	23 0.2	13.87	1.35	4,1	459....	0.0	23 23.2	11.96	0.23	2,3
396....	24.3	23 46.2	13.75	0.73	3,3	460....	0.1	24 1.2	14.25	0.49	4,5
398....	26.1	24 44.6	13.98	1.15	5,3	461....	0.1	23 26.0	14.13	0.25	3,4
400....	27.5	23 14.6	11.18	0.24	3,3	462....	0.3	23 9.4	11.56	0.17	2,3
401....	27.7	23 36.3	....	......	.,1	463....	0.7	23 17.0	13.43	0.37	3,4
401*a*....	28.1	23 46.8	15.08	1.46	3,3	464....	0.7	24 25.8	13.88	0.67	4,2
403....	28.4	24 21.0	11.40	0.18	2,3	465....	0.8	24 26.0	13.90	0.22	4,2
404*a*....	28.5	23 35.3	14.83	0.81	1,3	466....	1.4	24 25.4	14.51	0.93	3,2
405*a*....	28.7	24 16.6	15.28	0.49	4.1	467....	1.5	23 7.5	12.76	0.28	4.4
406....	29.3	24 37.0	15.06	1.39	5,4	468....	1.5	22 53.0	11.58	......	2,.
407....	29.4	24 11.9	13.82	0.48	3,2	469....	1.6	23 40.8	13.85	0.46	3,3
408....	29.8	23 19.2	13.43	1.24	3,2	470....	1.8	23 16.7	13.94	1.05	3,4
409....	30.4	23 13.9	11.92	1.22	3,3	471....	2.1	22 53.2	13.96	0.73	3,1
411....	31.2	24 14.8	12.30	1.70	4,3	473....	2.7	23 7.0	14.31	0.42	4,4
415....	33.5	23 38.6	13.89	0.96	4,3	475....	4.6	23 34.2	12.49	1.14	2,2
416....	34.0	24 14.9	14.56	0.60	4,2	476....	5.1	23 46.7	13.51	1.53	5,5
417....	34.2	24 46.2	14.32	0.76	4,4	477....	5.6	23 20.1	11.94	0.26	2.3
417*a*....	34.5	24 0.2	15.62	0.96	2,3	478....	6.0	24 17.4	14.60	0.36	4.2
418....	34.7	24 29.1	....	......	1,1	479....	7.0	24 33.4	11.28	−0.12	3,2
419....	34.8	24 17.7	14.76	0.51	4,2	481....	8.1	24 36.0	14.90	0.52	3,2
420....	35.6	24 22.5	13.92	0.96	4,3	482....	8.4	23 45.8	13.52	1.24	3,5
421....	36.2	23 39.4	14.14	0.28	4,3	483....	9.4	24 9.9	12.67	0.59	6,5
422....	36.4	24 11.6	13.80	1.27	4.3	484....	9.5	23 4.4	11.04	0.88	1,2
423....	37.0	23 22.2	13.66	0.49	3,3	485....	9.5	24 8.6	14.54	0.21	6,5
424....	37.9	24 13.6	12.00	1.66	4,3	487....	11.0	24 34.7	13 28	0.36	3,2
425....	38.7	23 54.3	13.57	1.07	5,4	488....	11.2	23 33.4	12.56	0.24	2,2
426....	40.9	22 58.5	13.56	0.57	4,2	489....	11.2	23 25.7	14.21	0.25	3,5
428....	41.6	24 46.4	13.33	0.77	3,3	490....	12.1	23 56.3	13.05	0.45	3,3
428*a*....	42.7	23 9.5	14.42	1.10	3,3	490*a*....	14.2	23 8.1	14.53	0.57	2,4
429....	43.1	24 46.5	13.58	0.90	4,3	491....	14.3	24 47.4	12.99	0.31	3,2
430....	43.1	24 3.3	11.27	0.39	5,4	492....	16.1	23 41.4	13.78	1.88	2,2
431....	43.2	24 22.6	12.98	0.40	4,3	494....	18.6	22 56.6	12.42	0.40	2,1
432....	43.4	23 41.6	11.85	1.06	4,3	495....	18.6	24 44.4	11.22	0.04	3,2
433....	43.6	23 50.4	12.53	0.83	4,4	496....	19.2	23 20.6	12.94	0.99	2,3
434....	44.1	23 49.7	12.58	0.86	4,4	497....	19.5	23 59.1	13.39	0.53	3,3
435....	45.0	23 39.8	14.31	1.02	4,3	498....	19.7	22 51.7	14.12	0.03	2,1
436....	46.0	23 58.3	12.63	0.66	5.5	498*a*....	19.9	23 20.8	14.48	0.57	2,3
437....	46.3	24 21.7	14.56	0.33	4,2	499....	20.8	23 35.0	11.05	0.49	1,2
438....	46.5	22 48.2	12.88	0.22	3,2	500....	21.1	23 19.7	....	......	.,2
440....	46.9	24 22.3	13.38	0.17	4,3	501....	21.3	24 32.3	11.72	0.12	5,3
441....	52.8	24 22.2	14.15	0.32	4.3	502....	21.6	24 28.0	13.54	0.50	5,4
442....	52.9	24 15.7	13.60	0.36	4,3	503....	21.8	24 22.0	13.85	0.40	3,2
443....	53.2	24 33.4	14.68	0.57	3,2	504....	22.5	23 45.2	13.08	1.16	3,5
444....	53.4	22 54.6	11.64	0.28	3,2	505....	23.6	23 24.8	13.43	0.25	3,5
445....	53.7	24 3.4	12.47	1.01	4,5	507....	25.7	24 32.2	13.65	1.13	3,2
446....	55.5	23 6.0	13.48	0.64	4,3	508....	26.2	24 34.2	11.04	0.12	3,2
447....	55.5	24 27.8	14.20	0.59	5,3	509....	27.1	23 17.0	13.30	0.23	2,3
448....	56.1	23 38.0	....	......	1,2	510....	27.4	22 57.6	13.99	0.55	2,1
449....	56.7	24 47.7	13.45	0.39	3,2	511....	28.0	24 41.2	11.84	0.52	3,1
450....	56.9	24 16.0	14.30	0.60	4,3	512....	29.1	23 27.0	13.87	0.79	4.5

TABLE II—*Continued*

Trumpler No.	R.A. 1900	Decl. 1900	Pg. Mag.	C.I.	No. of Plates	Trumpler No.	R.A. 1900	Decl. 1900	Pg. Mag.	C.I.	No. of Plates
	3^h42^m						3^h43^m				
513....	29.2	24° 0.0	12.34	0.11	3,3	575....	0.4	24° 44.9	14.41	0.58	4,3
514....	29.4	23 44.4	....	......	1,3	576....	0.9	23 21.6	13.98	0.38	2,4
515....	29.5	23 19.7	11.96	0.25	2,3	577....	1.7	23 57.6	14.35	0.85	4,5
516....	31.7	23 47.0	13.84	0.54	5,5	578....	3.1	23 34.7	13.18	1.39	4,3
517....	32.1	24 33.9	13.58	0.61	3,2	579....	3.3	24 24.4	13.74	1.05	2,1
519....	32.7	24 7.3	13.24	0.11	4,4	580....	3.6	22 53.7	12.40	0.76	4,2
520....	33.0	22 59.6	13.50	0.70	2,1	581....	4.0	23 30.0	14.63	1.11	3,3
521...	33.5	23 36.4	13.92	0.37	2,2	581*a*....	5.5	23 47.8	15.32	1.00	1,1
522....	33.8	23 58.0	12.39	0.94	3,3	582....	5.8	24 18.1	14.44	0.33	4,3
523....	34.8	24 21.0	13.18	0.22	3,2	583....	6.3	23 45.2	13.83	0.90	1,1
524....	35.6	24 45.5	11.24	0.10	3,2	584....	7.3	22 56.8	12.46	0.58	4,2
525....	35.7	23 18.5	13.94	0.99	2,3	585....	8.5	24 5.4	14.39	0.84	1,3
526....	36.6	23 36.1	13.42	1.04	2,2	586....	9.1	23 1.7	13.44	0.59	1,1
527....	37.2	23 46.3	14.25	0.97	3,5	586*a*....	9.8	23 43.9	14.49	0.00	1,1
529....	39.0	24 34.1	14.26	0.52	3,2	587....	9.8	24 10.3	....	......	.,1
530....	39.2	23 7.6	....	......	.,3	588....	9.9	23 28.5	11.03	0.70	1,1
531....	39.4	23 30.7	12.76	1.54	2,2	589....	10.0	23 20.8	13.89	0.97	1,3
532....	39.6	23 54.7	13.12	−0.03	3,3	590....	10.2	24 26.8	....	......	.,1
533....	39.6	24 28.4	14.47	0.99	4,3	591....	10.5	23 39.6	13.83	0.53	4,3
534....	41.4	24 22.5	11.96	0.08	3,2	592....	10.9	23 42.2	13.67	1.01	3,3
535....	42.7	24 8.8	12.34	0.45	4,4	592*b*....	11.2	24 1.0	14.49	0.05	1,2
536....	42.7	23 14.4	12.50	0.13	2,3	592*c*....	11.5	24 1.0	15.32	0.78	1,2
537....	42.8	24 27.9	13.66	0.40	4,3	593....	12.6	23 19.4	14.47	0.48	1,4
538....	43.2	23 0.7	13.40	0.63	2,1	595....	13.2	24 28.1	....	......	.,1
539....	43.4	24 37.4	14.12	0.43	3,2	596....	13.5	24 19.8	....	......	.,1
540....	43.4	23 28.2	13.18	1.30	3,5	597....	13.5	23 33.7	12.58	0.50	2,2
541....	44.8	22 59.5	12.54	0.27	2,1	598....	13.7	23 16.2	14.14	0.44	1.4
542....	45.2	24 1.4	12.26	0.99	3,3	600....	13.9	22 57.7	14.01	0.38	1,1
543....	45.2	23 28.4	13.76	0.50	3.5	601....	14.1	23 18.3	11.13	0.17	2,3
544....	45.6	24 10.5	13.96	0.18	3,2	603....	14.5	23 55.8	12.59	0.57	2,3
545....	45.9	23 31.6	14.27	0.37	2,2	604....	15.1	23 13.4	12.82	1.38	1,4
546....	46.0	24 0.9	14.08	0.97	3,3	605....	15.1	23 45.4	14.33	1.21	1,1
547....	46.8	24 23.7	14.15	0.33	3,2	606....	15.4	24 26.6	....	......	.,1
548....	47.7	23 34.8	13.40	0.26	2,2	608....	15.8	22 57.6	11.06	......	2,.
549....	48.6	24 1.2	13.29	0.36	3,3	609....	16.5	23 8.8	14.44	0.60	1,2
550....	48.7	22 52.9	14.24	1.01	2,1	610....	16.8	24 10.3	13.20	0.60	1,1
552....	49.1	23 50.4	14.18	0.46	3,3	612....	17.1	23 11.0	14.26	0.28	1,3
553....	49.4	24 27.4	14.52	0.50	3,2	612*a*....	18.2	24 2.2	15.51	0.87	1,2
554....	49.5	23 20.0	14.28	0.94	2,3	613*a*....	20.2	23 45.7	14.83	1.02	1,1
555....	49.6	22 53.2	14.23	0.30	3,1	614....	20.4	24 6.5	12.37	0.59	2,3
556....	50,0	24 14.2	14.35	0.49	3,2	614*a*....	21.1	23 50.2	14.90	0.40	1,2
557....	50.3	23 1.9	14.56	1.12	2,1	615....	21.9	23 56.8	12.96	0.82	2,3
558....	50.4	23 13.1	11.22	1.03	2,2	616....	22.0	24 28.2	13.47	0.89	2,3
559....	50.5	24 43.0	12.84	0.34	4,3	617....	23.3	24 43.4	13.88	0.35	2,2
560....	50.8	24 19.6	....	......	.,3	618....	23.3	23 43.2	12.87	0.25	2,1
561....	51.0	24 25.1	14.40	0.50	3,2	619....	23.4	23 57.5	14.01	0.44	2,3
562....	51.6	23 57.6	11.75	0.59	2,5	620....	23.5	22 59.0	13.97	1.01	2,1
563....	52.7	23 40.2	13.54	0.76	2,3	621....	23.5	24 38.1	14.92	0.24	2,2
564....	52.7	23 7.0	13.34	0.33	1,2	621*a*....	23.7	23 50.8	15.04	......	1,.
565....	53.0	24 28.1	14.37	0.63	2,1	623....	24.2	23 17.3	1[illegible].66	1.14	1,3
566....	53.2	24 44.2	14.88	1.00	4,3	624....	26.0	24 33.8	13.98	0.30	2,2
567....	54.2	23 59.0	13.86	0.44	3,5	625....	26.4	24 28.6	13.84	0.70	3,3
568....	55.5	24 28.0	14.65	0.75	2,1	626....	26.7	24 37.9	11.54	0.88	2,2
569....	56.4	24 24.3	12.85	......	3,.	627....	27.1	24 25.1	13.81	0.75	1,1
569*a*....	57.6	23 34.2	15.50	0.91	1,3	628....	27.6	23 32.0	12.07	1.02	2,1
570....	58.1	23 50.0	12.14	0.52	3,4	630....	28.7	23 10.2	13.66	0.84	1,3
571....	58.3	23 27.6	12.86	0.00	1,2	632....	28.9	23 29.3	14.14	2.19	2,1
572....	58.3	23 31.6	12.98	0.41	3,3	633....	29.4	23 30.2	12.76	1.22	2,1
572*a*....	58.7	23 29.1	15.02	0.66	1,2	634....	30.4	22 50.8	14.16	0.61	2,1
573....	59.4	23 59.0	12.55	1.19	2,2	635....	30.7	24 0.2	13.28	0.74	2,3

TABLE II—*Continued*

Trumpler No.	R.A. 1900	Decl. 1900	Pg. Mag.	C.I.	No. of Plates
	3^h43^m				
636....	31ˢ.4	22°48′.4	13.38	0.58	2,1
637....	32.4	22 55.4	14.27	0.53	2,1
638....	32.6	24 10.0	13.89	0.47	1,2
639....	32.6	24 4.7	14.04	0.42	2,2
640....	32.6	23 24.4	12.13	0.74	1,2
641....	32.7	23 34.7	13.82	2.09	2,1
642....	32.7	23 57.8	13.99	0.43	2,2
643....	33.3	24 10.4	14.41	0.39	1,2
644....	33.6	24 29.0	14.24	0.69	3,3
645....	34.9	24 21.0	13.53	0.53	1,1
646....	35.2	24 19.4	11.40	......	1,.
647....	35.9	22 58.4	13.69	0.69	2,1
648....	36.7	24 20.4	13.57	0.51	1,1
649....	36.8	23 29.4	11.47	0.90	2,1
650....	36.8	23 43.5	13.68	0.62	2,1
650a....	36.9	24 3.6	14.66	0.05	2,2
652....	38.2	23 46.4	13.74	0.53	4,3
653....	38.6	24 35.3	14.58	1.38	2,2
654....	38.7	24 11.9	13.75	0.23	1,1
655....	38.8	24 44.6	14.64	0.48	2,2
656....	38.9	23 57.2	13.59	0.36	2,2
657....	39.0	23 59.4	12.52	0.98	2,3
657a....	40.5	23 27.2	15.06	0.54	2,1
658....	40.6	23 9.8	13.81	0.15	1,2
659....	41.0	23 10.8	13.60	0.54	1,1
660....	41.2	24 22.6	13.89	0.73	1,1
661....	42.0	23 55.3	13.32	1.14	2,2
662....	42.3	23 39.0	13.88	0.76	2,1
663....	43.6	24 7.6	13.74	0.15	3,3
664....	43.8	24 6.6	12.62	0.65	3,3
665....	43.8	23 43.9	11.96	1.32	2,1
666....	44.0	23 8.6	14.19	0.33	1,1
667....	44.2	22 59.1	12.02	0.63	2,1
668....	44.4	24 9.5	13.01	0.83	3,3
669....	44.6	23 22.6	14.26	0.89	1,2
672....	47.9	24 2.6	12.96	0.88	2,2
673....	48.1	24 13.3	14.00	0.29	1,1
674....	48.6	22 58.0	12.28	0.18	2,1
675....	48.6	23 56.4	11.11	1.04	2,1
676....	49.2	24 41.4	13.94	1.08	2,2
677....	49.5	23 28.5	11.70	0.35	2,3
678....	50.0	23 26.4	11.93	0.67	3,3
679....	50.7	23 49.6	13.88	0.40	4,2
680....	54.2	23 24.0	12.30	0.57	2,2
681....	54.4	24 12.2	13.05	0.41	1,1
682....	54.6	23 59.8	13.44	0.39	2,2
683....	55.5	23 25.3	14.98	0.50	2,2
684....	56.6	23 43.4	13.16	0.56	2,1
685....	57.0	24 2.2	13.23	0.69	2,2
687....	57.6	23 13.5	11.50	......	1,1
690....	58.9	24 17.0	13.20	0.24	1,1
691....	59.8	23 32.6	14.67	......	1,.
	3^h44^m				
692....	0.4	23 38.7	14.04	0.88	2,1
696....	1.4	22 54.8	......	......	.,1
698....	2.7	24 3.3	12.66	0.36	2,2
699....	3.7	23 36.6	13.80	0.37	2,1
700....	3.8	23 53.0	14.01	0.57	1,2
	3^h44^m				
701....	4ˢ.6	23° 6′.7	14.00	0.94	2,3
702....	5.2	24 23.6	13.60	0.67	1,1
703....	6.0	24 22.0	12.41	0.46	1,1
704....	6.2	24 19.2	13.47	0.63	1,1
705....	8.8	23 54.6	11.98	0.48	2,2
706....	10.0	23 59.6	14.42	0.55	2,3
707....	10.1	23 42.8	13.52	1.03	2,1
708....	11.1	23 49.1	12.30	1.20	3,2
709....	12.6	23 50.4	14.33	0.85	2,2
710....	12.8	23 55.4	12.49	1.28	3,3
711....	13.3	24 36.3	13.90	0.67	4,3
712....	14.2	24 13.7	14.28	1.12	2,1
713....	14.4	23 59.5	14.46	1.10	3,3
714....	14.7	24 19.4	11.32	0.04	2,2
715....	14.7	24 9.9	13.78	0.35	1,1
716....	15.0	22 54.1	14.03	0.52	4,2
717....	15.1	24 29.2	13.90	0.50	2,1
718....	16.0	23 30.9	13.77	0.41	3,2
719....	16.6	24 30.1	14.14	0.52	2,1
720....	17.2	23 34.8	13.93	0.48	4,2
722....	18.0	23 34.8	13.69	0.21	4,2
723....	18.9	23 11.6	14.28	0.56	2,3
725....	19.9	24 46.7	14.40	0.22	5,2
726....	22.8	24 10.2	12.92	0.15	2,1
727....	22.9	24 9.8	11.74	0.86	2,2
728....	23.4	24 0.2	14.54	0.48	3,3
729....	23.4	23 36.1	14.48	0.58	4,2
730....	23.5	23 34.6	13.12	0.24	4,2
731....	23.5	23 38.9	13.02	1.40	4,2
732....	23.6	24 17.8	14.44	0.34	1,2
733....	23.7	23 30.2	14.08	0.48	3,2
734....	24.8	23 55.0	14.22	1.00	3,3
735....	25.2	22 50.6	14.24	0.56	4,2
736....	26.6	23 52.6	14.47	0.72	3,3
736a....	27.0	23 3.8	14.86	0.39	2,1
737....	27.1	23 34.2	14.46	0.22	4,2
738....	27.9	24 27.4	13.70	1.14	1,1
739....	28.8	23 52.9	14.66	0.55	2,3
740....	29.1	23 0.4	13.85	0.80	4,2
741....	29.1	24 21.2	14.47	1.12	2,2
742....	29.9	24 11.5	......	......	.,2
743....	30.2	24 2.6	12.67	1.24	3,3
744....	30.5	24 12.8	11.58	0.75	2,2
745....	30.6	23 16.1	11.58	0.38	2,3
746....	31.8	23 35.2	13.98	0.36	4,2
747....	32.2	24 32.0	12.54	0.40	4,2
748....	32.3	23 43.8	13.81	......	3,.
749....	33.5	24 12.3	11.34	0.27	2,2
750....	33.7	24 13.5	14.12	0.96	1,2
752....	35.1	24 33.9	14.20	0.48	5,2
753....	35.1	23 52.6	13.72	0.40	2,2
755....	35.8	24 36.1	11.76	0.49	4,4
757....	36.1	23 57.0	13.56	0.27	2,3
758....	36.3	24 2.6	10.92	0.40	1,1
759....	36.3	23 0.2	14.30	0.86	3,2
760....	36.4	24 12.3	13.96	1.00	2,1
761....	37.9	22 57.1	13.84	0.54	3,1
762....	39.3	23 58.0	14.08	0.63	2,3
763....	39.3	23 29.2	14.75	0.50	2,1
764....	39.9	24 14.4	14.04	0.31	2,1

TABLE II—*Continued*

Trumpler No.	R.A. 1900	Decl. 1900	Pg. Mag.	C.I.	No. of Plates
	3^h44^m				
765....	40ˢ.0	24°37′.1	15.01	0.89	4,2
766....	40.5	23 11.4	13.32	1.37	2,2
767....	40.8	23 39.4	14.60	0.95	3,1
768....	41.2	22 56.4	13.54	0.37	3,1
769....	41.4	23 53.5	12.42	0.37	2,1
770....	41.6	23 34.3	13.32	0.52	3,1
771....	42.2	22 55.0	13.62	0.55	3,1
772....	43.3	23 37.8	10.63	......	1,.
773....	44.1	24 17.0	14.16	0.83	2,1
774....	44.6	24 43.0	12.34	1.09	4,2
775....	46.2	23 15.5	14.26	0.41	2,2
776....	46.4	23 2.1	14.21	0.68	2,1
777....	46.6	23 0.4	12.18	1.12	3,1
778....	46.8	23 33.2	14.52	0.96	3,1
780....	48.5	22 56.4	13.77	0.63	4,2
781....	48.6	23 4.8	14.10	1.02	3,2
782....	50.0	24 8.2	13.71	0.39	2,2
783....	50.3	24 23.9	14.22	0.49	1,1
783a....	50.4	23 12.0	15.20	0.51	1,1
784....	51.0	24 7.6	11.42	1.06	2,2
785....	51.4	24 16.8	13.41	0.34	1,1
786....	51.8	23 33.1	13.20	0.24	2,1
787....	53.6	24 24.3	14.03	1.12	1,1
788....	53.8	23 7.2	11.77	0.35	2,2
789....	54.5	23 25.4	13.46	0.32	2,2
790....	54.6	23 26.3	14.62	0.86	3,2
792....	55.7	24 29.1	14.34	0.53	4,3
794....	57.0	23 1.3	14.44	0.68	2,1
795....	57.6	23 53.7	12.26	0.58	1,1
796....	57.7	23 31.9	12.20	0.73	2,1
797....	57.9	23 19.2	13.65	0.31	1,2
798....	58.7	23 10.7	13.69	0.46	1,1
799....	59.0	24 21.8	14.52	0.76	1,1
800....	59.1	24 32.6	12.70	0.52	3,2
801....	59.1	24 14.7	13.35	0.39	1,1
803....	59.3	23 56.9	14.07	0.71	1,1
804....	59.9	24 8.8	13.92	0.52	2,2
	3^h45^m				
805....	0.3	23 26.0	14.20	0.74	3,2
806....	0.5	24 22.6	13.45	0.49	1,1
807....	2.8	24 45.2	14.00	0.82	3,2
808....	4.0	24 23.8	14.22	0.89	1,1
809....	4.7	24 2.4	13.65	0.35	1,1
810....	4.8	23 55.2	14.12	0.70	1,1
811....	5.4	24 17.7	14.12	0.39	1,1
812....	5.5	23 36.1	11.18	......	2,.
813....	5.7	24 38.2	12.46	0.98	3,2
814....	5.9	24 39.2	14.48	0.94	3,2
815....	6.1	23 39.3	13.34	1.29	2,1
816....	6.3	23 54.0	14.00	0.70	1,1
817....	6.8	24 26.1	14.74	0.93	1,1
819....	7.2	24 22.3	12.13	1.10	1,1
820....	7.5	23 48.5	13.79	1.02	3,1
821....	9.8	24 3.3	14.03	1.29	1,1
822....	9.8	24 45.2	14.39	0.95	3,2
823....	11.0	23 35.7	13.62	0.39	2,1
824....	12.3	23 55.6	14.00	0.15	1,1
825....	12.9	24 26.8	13.70	0.56	2,2
	3^h45^m				
827....	15ˢ.4	23°20′.8	13.17	0.14	1,1
829....	15.6	24 13.0	12.54	0.15	1,1
830....	17.1	22 59.1	14.27	0.46	2,1
831....	17.3	23 12.7	13.14	0.45	1,1
833....	20.2	23 31.5	13.27	0.64	2,1
834....	20.3	24 17.2	13.41	0.38	1,1
835....	21.5	23 55.1	14.44	0.54	1,1
835a....	22.	23 22.6	14.74	1.38	1,1
836....	22.3	24 4.8	11.43	......	1,1
837....	22.3	24 32.0	14.48	0.90	3,2
838....	25.0	24 33.6	12.11	0.86	3,2
839....	25.0	24 32.0	14.80	0.88	3,2
840....	25.3	24 13.4	14.19	0.83	1,1
841....	25.7	24 21.8	13.45	0.49	1,1
842....	25.9	24 33.5	14.10	0.66	3,2
843....	27.1	23 23.6	13.69	0.39	1,1
844....	27.1	23 47.2	14.38	1.02	3,1
845....	27.2	24 46.5	11.69	0.49	3,2
846....	28.3	23 28.9	14.33	1.13	2,1
847....	28.5	23 39.4	13.14	0.98	2,1
849....	29.1	24 25.0	14.22	0.49	1,1
850....	29.5	24 15.7	12.23	0.45	1,1
851....	29.6	24 4.5	14.54	0.43	2,1
853....	30.3	24 42.8	13.96	0.50	2,1
854....	30.5	23 21.1	14.22	......	1,.
855....	30.6	23 33.4	14.36	0.71	2,1
856....	31.0	24 30.3	14.50	0.24	3,2
857....	31.1	23 37.4	12.54	0.69	2,1
859....	32.0	22 57.9	11.62	0.59	2,1
861....	33.0	23 54.3	14.44	0.63	1,1
862....	33.1	23 35.9	14.71	1.01	2,1
863....	33.9	24 19.0	14.22	1.17	1,1
865....	35.0	24 11.1	14.25	0.40	1,1
866....	35.2	24 18.6	13.50	0.40	1,1
867....	35.2	23 43.9	12.90	......	2,.
868....	35.4	23 20.1	13.69	0.33	1,1
869....	35.5	23 5.8	14.69	0.96	2,2
870....	35.6	23 36.6	12.66	0.64	2,1
871....	35.7	23 49.8	13.73	0.37	1,1
872....	35.9	24 41.9	14.05	0.65	3,2
873....	37.2	23 42.8	14.68	......	2,.
874....	39.2	24 43.6	14.24	0.48	3,1
875....	39.5	23 44.7	14.55	......	2,.
877....	40.2	24 46.1	14.66	0.62	3,2
878....	40.6	24 10.5	14.22	0.57	1,1
879....	40.9	24 14.9	12.73	0.49	1,1
880....	41.0	24 40.9	11.61	0.57	3,2
881....	41.7	23 27.5	14.59	0.68	2,2
882....	42.1	23 42.6	14.64	......	2,.
883....	42.2	24 22.0	12.73	0.60	1,1
884....	42.5	23 20.4	14.22	0.61	1,1
885....	42.8	24 40.2	12.45	0.43	3,2
886....	42.8	22 52.9	14.26	0.93	2,1
887....	44.0	24 40.4	13.68	0.34	3,2
888....	44.9	24 15.0	12.90	0.10	1,1

TABLE II—*Continued*

Trumpler No.	R.A. 1900	Decl. 1900	Pg. Mag.	C.I.	No. of Plates	Trumpler No.	R.A. 1900	Decl. 1900	Pg. Mag.	C.I.	No. of Plates
	3^h45^m						3^h45^m				
890....	46ˢ8	23°13ʹ4	14.52	0.76	1,1	903....	51ˢ2	23°41ʹ1	14.33	......	2,.
892....	47.4	24 25.5	13.14	1.73	1,1	905....	52.1	24 25.7	14.00	......	1,.
894....	48.0	22 53.4	12.69	0.49	2,1	906....	52.7	24 41.2	13.90	0.28	3,2
895....	49.4	24 46.0	13.84	0.92	3,2	907....	53.2	24 36.5	11.41	0.06	3,2
896....	49.7	24 19.4	13.45	0.15	1,1	908....	53.4	23 19.4	14.12	0.44	1,1
897....	49.7	24 3.6	14.19	0.83	1,1	909....	54.4	24 44.2	11.47	0.21	3,2
898....	49.8	23 55.4	14.07	0.24	1,1	910....	55.0	23 56.0	12.85	0.01	1,1
899....	50.1	22 53.9	12.76	0.33	2,1	915....	57.3	24 23.0	14.22	......	1,.
901....	50.9	24 46.3	13.37	0.47	3,2	......	......	......	......	......	......
902....	51.0	23 27.0	......	......	.,1	......	......	......	......	......	......

MOUNT WILSON OBSERVATORY
March 1921

ON MAJORANA'S THEORY OF GRAVITATION[1]

By HENRY NORRIS RUSSELL[2]

ABSTRACT

Majorana's theory of gravitation.—As a result of some delicate pendulum experiments, Majorana has suggested that gravitational force is weakened by passing through matter. In this paper certain *astronomical consequences* of this assumption are presented. The true masses of the planets according to this theory are computed and it is shown (1) that the inertial masses cannot be equal to these true masses, and (2) that if we assume them equal to the apparent gravitational masses we are led to such discrepancies in the case of the tides that we are forced to conclude that the absorption of gravitational force cannot exceed 1/5000 of the value assigned by Majorana.

Possible influence of one body on the mass of another body.—The interpretation of Majorana's positive experimental result, assuming it to be real, must be that the mass of a body is actually changed by the presence of another body.

Professor Majorana, in a very interesting series of papers,[3] has proposed a new theory of gravitation, with experimental evidence supporting it. The present communication deals with certain astronomical consequences of the theory, in the form in which he presents it, and suggests a modification which will remove the resulting discrepancies.

1. The law which Majorana proposes[4] is that the force of attraction between any two material particles is directed along the line joining them, but that its intensity is less than that given by Newton's law whenever this line traverses matter, being diminished in accordance with the "law of progressive absorption" which holds good for radiation.

If M_1 and M_2 are the masses of the particles, r their distance apart, and k the ordinary constant of gravitation, the attractive force will be

$$f=k\frac{M_1M_2}{r^2}e^{-\int h\theta dr}, \tag{1}$$

[1] *Contributions from the Mount Wilson Observatory*, No. 216.

[2] Research Associate of the Mount Wilson Observatory.

[3] *Philosophical Magazine*, **39**, 488, 1920. *Atti della Reale Accademia dei Lincei*, **28**, 160, 221, 313, 416, and 480, 1919, and **29**, 23, 90, 163, and 235, 1920. Further citations refer to the detailed account in Italian.

[4] *Op. cit.*, p. 170.

where h is a second universal constant, that of absorption of gravitation, and θ the density of matter at any point on the line joining the particles, and the integral is taken over the whole length of this line.

In consequence of this absorption the attraction of any large body upon objects outside it will be diminished, and its "apparent mass," measured by means of its attraction, will be less than the true mass. For spherical bodies of uniform density Majorana shows[1] that the attraction will still be exactly proportional to the inverse square of the distance, while the apparent mass m will be connected with the true mass M by the equations

$$m=\psi M, \tag{2}$$

$$\psi=\tfrac{3}{4}\left[\frac{1}{p}-\frac{1}{2p^3}+e^{-2p}\left(\frac{1}{p^2}+\frac{1}{2p^3}\right)\right], \tag{3}$$

$$p=hR\theta, \tag{4}$$

where R is the radius of the mass, and θ the *true* density.

For small values of p we find, expanding in series,

$$\psi=1-\tfrac{3}{4}p+\tfrac{3}{5}p^2-\tfrac{1}{6}p^3+\ .\ .\ .\ .\ . \tag{5}$$

For large values of p, ψ approaches asymptotically to the value $3/4p$ and m to the value $\pi R^2/h$. The apparent mass of a homogeneous sphere has therefore, according to this theory, a superior limit, depending only upon its superficial area and the universal constant h. Majorana gives no calculations for bodies not of uniform density; but an easy calculation shows that the attraction of any spherical mass in which the density depends only upon the distance from the center, upon an external particle, is inversely proportional to the square of the distance from the center, as in the Newtonian case. The difference between the true and apparent masses will be greater, the greater the central condensation. For example, if the density at the distance r from the center is proportional to $(R^2-r^2)^n$, it is easy to show[2] that

$$\psi=1-\frac{(2n+3)^2}{12n+12}p+\ .\ .\ .\ .$$

[1] *Op. cit.*, p. 420. The notation is slightly changed here.

[2] See the appendix to this paper.

(where the *true mean density* is to be used in calculating p). For $n=1$, $\psi=1-\frac{25}{24}p$ . . . . ; for $n=2$, $\psi=1-\frac{49}{36}p$. . . . . The former value corresponds to a central condensation greater than in the case of the earth, and the latter to a central density more than four times the mean density.

Majorana's conclusion, based upon qualitative reasoning, that condensation toward the center will not radically change the amount of absorption of gravitation, is therefore justified; but the numerical effects of such condensation appear to be somewhat greater than he has estimated.

2. We may now proceed to calculate the effects of absorption of gravitation in various bodies belonging to the solar system, using

TABLE I

GRAVITATIONAL ABSORPTION ACCORDING TO MAJORANA

	R	p	$p\psi$	ψ
Sun	6.95×10^{10} cm	1.41	0.660	0.33
Jupiter	7.23×10^{9}	1.33	0.065	0.951
Saturn	5.90×10^{9}	0.72	0.029	0.978
Earth	6.37×10^{8}	5.53	0.024	0.981
Mars	3.39×10^{8}	3.95	0.0090	0.993
Moon	1.74×10^{8}	3.40	0.0040	0.997
Eros	2×10^{6}	3	0.00004	1.0000

the value determined by Majorana[1] for the constant of absorption ($h=6.73\times10^{-12}$ in c.g.s. units) and assuming the bodies to be of uniform density. It must be remembered that, if ρ is the mean density derived from observation on the Newtonian theory, we will have $p=\psi\theta$, and hence $p\psi=hR\rho$. Knowing this, ψ may be determined by successive approximations, or, if p is small, by the equation

$$\psi=1-\tfrac{3}{4}p\psi-\tfrac{13}{80}p^2\psi^2-\tfrac{53}{480}p^3\psi^3\ .\ .\ .\ . \qquad (6)$$

obtained by "reversion" from (5).

The results are given in Table I. The data for the sun are from Majorana's paper. The diameter and density given for the asteroid Eros are rough estimates, but doubtless correct as regards order of magnitude.

[1] *Op. cit.*, **29**, 236.

It is evident that the absorption of gravitation within the larger planets is by no means negligible.

3. The changes which such an absorption would introduce into the planetary theory are remarkable. The attraction between two spherical bodies of masses M_1 and M_2 at a distance r is kM_1M_2/r^2 according to Newton. According to Majorana, we must substitute $kM_1M_2\psi_1\psi_2/r^2$ (since the force is weakened in traversing both bodies).

Nothing in Majorana's discussion suggests that any change would occur in the inertial masses. If these remain the same, the acceleration of M_1 will be $kM_2\psi_1\psi_2/r^2$, or $km_2\psi_1/r^2$. Similarly that of M_2 will be $km_1\psi_2/r^2$, and the relative acceleration of the bodies toward one another will be $k(m_1\psi_2+m_2\psi_1)/r^2$.

The expression in parenthesis takes the place of the sum of the masses, M_1+M_2, in the Newtonian theory. Let M_1 represent the sun and M_2 the planet. We will have, by Kepler's Third Law,

$$\frac{a_2^3}{T_2^2}=\frac{k}{4\pi^2}(m_1\psi_2+m_2\psi_1), \tag{7}$$

where a_2 and T_2 are the mean distance and periodic time of the planet. Comparing this with another planet, M_3, we find:

$$\left(\frac{a_2}{a_3}\right)^3=\left(\frac{T_2}{T_3}\right)^2\frac{m_1\psi_2+m_2\psi_1}{m_1\psi_3+m_3\psi_1}.$$

The masses of the planets are all less than a thousandth part of the sun's, and the factor ψ_1 is also small. We may therefore write with high approximation:

$$\frac{a_2}{a_3}=\left(\frac{T_2}{T_3}\right)^{\frac{2}{3}}\left(\frac{\psi_2}{\psi_3}\right)^{\frac{1}{3}}.$$

The error committed by disregarding the planetary masses is at most one part in ten thousand. This equation differs from the ordinary form of Kepler's law by the presence of the factor involving ψ_1 and indicates that a massive planet should be considerably nearer the sun than a planet of small mass and identical period.

Applying it first to Jupiter and the earth, we have $\psi_2/\psi_3=0.969$, so that, taking the earth's mean distance as unit, the mean distance

of Jupiter should be less than that computed on the ordinary theory by 1.04 per cent.

This is utterly inadmissible. When Jupiter is in quadrature with the sun, its annual parallax (the angle at the planet between lines drawn to the sun and earth) is approximately $11° 5'$, and the suggested change would increase this angle by $6' 56''$. The planet's longitude would then deviate from the ordinary ephemerides, which are computed on the Newtonian theory, by this amount every time that Jupiter was in quadrature, but in opposite directions according as it was east or west of the sun. Any actual deviation of this sort cannot at most exceed 1/500 of the predicted amount, and to avoid conflict with observation, Majorana's constant h would have to be correspondingly diminished.

The case of Eros is even more striking. Taking the earth again as standard, we find that $\psi_2/\psi_3 = 1.019$, and the distance of the asteroid should be greater by 0.63 per cent. When it is in perihelion and at the same time in quadrature, its distance from the sun is 1.13 astronomical units and from the earth 0.55. The suggested change in its mean distance would displace it by $39'$ in geocentric longitude. With the planet in perihelion, but nearer to opposition (in about the position which it occupied in 1901, when it was very carefully observed), the discordance would exceed $1°$—which is fully five thousand times as much as is allowable.

The most conspicuous case of all is provided by the moon. On Majorana's theory, the attraction of the sun should produce in the moon an acceleration greater by 1.6 per cent than that experienced by the earth at the same distance. This would be equivalent to a continually acting disturbing force, directed toward the sun, and amounting to 1/60 of its whole attraction on the moon, or 1/30 of the earth's attraction. This is three times the maximum disturbing force due to the sun on the Newtonian theory, and its introduction would play utter havoc with the whole lunar theory. The admissible magnitude of such an unrecognized disturbing force is probably less than one ten-thousandth as great.

Yet again, the tides on the earth's surface would be radically altered. In addition to the usual tidal forces depending on the

variations in the magnitude and direction of the attraction of the sun and moon, there would be a force depending on the difference of the screening action for the earth as a whole and for a water particle on its surface. Over the hemisphere facing the sun this force would be directed toward the sun and amount to 1.9 per cent of the sun's attraction. In the opposite hemisphere it would rapidly diminish, changing sign for points such that the absorption along the "gravitational ray" from the sun was equal to the average value for the earth as a whole. Beyond this the force would be directed away from the sun and reach a maximum at the point opposite it, where it would be 2.9 per cent of the sun's whole attraction (assuming, as elsewhere, a homogeneous earth).

The tides are produced by the horizontal component of this force, which will evidently be a maximum when the sun (or moon) is on the horizon, and amount to 0.019 times the whole attraction of the body. The ordinary tide-raising force vanishes when the sun is on the horizon, is greatest when it is 45° above or below it, and amounts to 0.025 of the moon's whole attraction and 0.000064 of the sun's.

The additional tide-raising force demanded by Majorana's theory is therefore about three-quarters of the recognized force due to the moon, but nearly three hundred times the recognized component in the case of the sun. This last term is more than eighty times the greatest tidal force that can occur on the Newtonian theory.

4. We are forced therefore to the conclusion that upon the hypothesis that there exists an absorption of gravitational force in matter, without change in its inertial mass, the coefficient of absorption cannot exceed one ten-thousandth of that derived by Majorana from his experiments, and must be hopelessly beyond the reach of investigation in the laboratory. There is nothing really new in this conclusion, or in the reasoning by which it has been reached. All depends upon the old and familiar proposition that the motions of the planets prove that their gravitational and inertial masses are strictly proportional to one another, at least within a few parts in a million. Any influence which modifies one must alter the other in the same proportion.

Accepting this, we must assume in section 3 that the accelerations are inversely proportional to the apparent masses, m, instead of the true masses, M. It would then follow that all astronomical motions will take place in accordance with the Newtonian law, but that the apparent masses will everywhere appear in the place of the true masses. The only exceptions occur when one body intervenes directly between two others. The resulting effects are negligible for the planets and small for the moon; but in the case of the tides they are important.

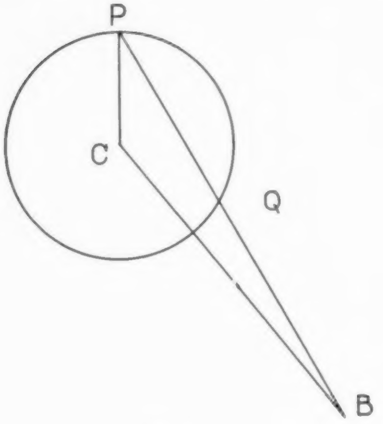


Fig. 1

Let the apparent mass of the earth be m_2 and the radius R, and let there be a body B (Fig. 1) of apparent mass m_1, at a distance D from the earth's center C, and D_1 from a point P on its surface. At this point there is a particle of apparent mass μ. The attraction of the earth upon it is

$$g=km_2\mu/R^2.$$

The horizontal tide-raising force acting upon P will be the sum of the components in this direction of the direct attraction of B and the inertial reaction arising from the acceleration of the system of axes, fixed in the earth, with respect to which the force is measured. (We may suppose that the earth does not rotate.) The force due to the attraction of B is:

$$f=\frac{km_1\mu}{D_1^2}\sin z_1 e^{-h\theta S},$$

where z_1 is the apparent zenith distance of B for an observer at P, S the distance PQ which the line PB traverses inside the earth, and θ the earth's true density (supposed as usual to be constant).

Since $S=-2\ R\cos z_1$, we have by (4):

$$f_1=\frac{km_1\mu}{D_1^2}\sin z_1\, e^{2p\cos z_1}=\frac{km_1\mu}{D_1^2}\sin z_1\,(1+2p\cos z_1)$$

(since p is small).

If $\cos z_1$ is positive, the last factor is to be set equal to unity.

In this case, where the attraction between P and B varies with their direction as well as their distance, the apparent gravitational mass of P will vary according to whether we consider the attraction between it and B, or between it and the earth. (We obviously cannot suppose that the apparent masses of the earth or of B, whose attraction on one another is unaltered, are sensibly affected by the existence of the minute particle P.)

The inertial mass of P may be equal to either of these values of its gravitational mass, or, more probably, to some compromise between them. Let us set it equal to $\mu\ (1-\epsilon)$. The acceleration of the earth, due to the attraction of B, will be $km_1/D^2 \sin z$, where z is the angle PCB (the geocentric zenith distance). We have, rigorously,

$$D \sin z = D_1 \sin z_1\,,$$

and, neglecting R^2/D^2,

$$D = D_1 + R \cos z_1\,.$$

The inertial reaction of the particle P, in the horizontal direction, will be, to the first order:

$$f' = \frac{km_1\mu}{D^2}(1-\epsilon)\sin z = \frac{km_1\mu}{D_1^2}\sin z_1\left(1-\epsilon-\frac{3R}{D}\cos z_1\ \ldots\ldots\right).$$

The deflection of the vertical by the tidal force will be given by

$$\tan\delta = \frac{f_1 - f'}{g} = \frac{m_1}{m_2}\frac{R^2}{D_1^2}\sin z_1\left(3\frac{R}{D}\cos z_1 + 2p\cos z_1 + \epsilon\right).$$

The first term in the parenthesis gives the effect on the Newtonian theory.

The quantity ϵ must be very small. If the inertial mass of the pendulum, swinging under the earth's attraction, were $1-\epsilon$ times its gravitational mass, its period would be $1-\epsilon/2$ times that derived from the ordinary theory. When the sun and moon are above the horizon, ϵ must be zero. If it were as great as 0.001 when both are below the horizon, all pendulum clocks would gain at the rate of nearly two seconds an hour. The greatest admissible value must be well below 0.0001. On the other hand, $3R/D$ is 0.050 in the case of the moon, and 0.00013 for the sun, while $2p$, for the earth, is 0.048.

The assumption of an absorption of gravitational force in passing through the earth leads therefore to discrepancies in the case of the tides which cannot be removed by any admissible assumption regarding changes in the inertial mass of the attracted water. The lunar tides should be twice as great, and the solar tides 370 times as great, when the attracting bodies are below the horizon as when they are at the same altitude above.

The most accurate observations of tidal force are undoubtedly those of Michelson and Gale, derived from observations of the displacements of the water level in underground pipes. From their curves[1] it is evident that the Newtonian theory represents the forms of the observed curves within a few per cent. The relative amplitudes of the solar and lunar tides, for a whole year, agree with theory within the probable errors (about 3 per cent for the larger oscillations). This indicates that any absorption of gravitation, in passing through the earth, cannot exceed 1/5000 of the value assigned by Majorana.

5. It appears therefore that the assumption that gravitational force is weakened in passing through matter must be definitely abandoned.

But what then becomes of Professor Majorana's long and careful series of experiments? If their result is accepted, it seems necessary to interpret it as showing that the *mass* of one body (his suspended sphere of lead) was diminished by the presence of another large mass (the surrounding mercury); that the effect was a true change in the mass (since inertial mass and gravitational mass are all the kinds of mass that we know of); and that it depended on the proximity of the larger mass, and not upon any screening action upon the earth's gravitation.

Strange as this notion may seem, it is not inherently absurd. Indeed, if the phenomena of gravitation and inertia may be accounted for by assuming that the four-dimensional "world" possesses certain non-Euclidean properties, or "curvature," both in the presence of matter and remote from it, it is not very surprising if the curvature induced by one mass of matter should be

[1] *Astrophysical Journal*, **50**, 342, 1919.

modified to some degree by the superposition of the curvature due to another, so that the effects were not exactly additive.

A great variety of assumptions may be made regarding such an influence, and many of these might have the advantage of giving a conservative field of force, which Majorana's did not. Complications are still likely to arise. For example, consider a large spherical mass, alone in space and gradually contracting upon itself while it moves forward in a straight line. Its mass will presumably diminish as its various parts come closer together; but what will happen to its velocity? Presumably this would increase, but it seems obvious that either the conservation of momentum or the conservation of energy would have to be abandoned, if not both. Come next to a planet revolving in an eccentric orbit about the sun. Its mass will diminish at perihelion, and this will probably lead to changes in its orbital velocity. The resulting alterations in the orbit will depend on the law of change of velocity, and it might be possible to invent a law which would lead to conclusions consistent with observation.

Further speculation on such matters seems, however, to be premature, when it is considered that the whole structure would rest upon the observation of a change in weight amounting to one part in 1200 millions. Discussion of the possibility that some undetected systematic error has crept into the results, in spite of the great care taken to eliminate such errors, or to correct them, must be left to those better versed in experimental technique than the present writer; but it is to be hoped that the further experiments which Professor Majorana contemplates will provide the data regarding the reality, magnitude, and laws of variation of the suspected influence which are now evidently desirable.

APPENDIX

The proofs of certain statements made previously are collected here to avoid interruption of the course of the discussion.

Let P, Figure 2, be any point at which it is desired to find the gravitational attraction of a mass M. Draw through P an elementary cone, of solid angle $d\omega$, intersecting M between the points

A and B. Let Q be any point inside M at a distance r from P. The element of this cone between the distance r and $r+dr$ from P will have a volume $r^2 dr d\omega$ and a mass $\theta r^2 dr d\omega$, where θ is the true density. Its attraction upon a unit particle at P will be $k\theta dr d\omega$, according to Newton's law. If we set $\theta dr = dn$, n will be the total mass per unit section of a thin cylindrical column extending from Q to A, and the attraction at P will be $k d\omega dn$. Integrating, we find for the attraction of the conical frustum AB the value $kn_1 d\omega$ where n_1 is the value of $\int \theta dr$ between A and B.

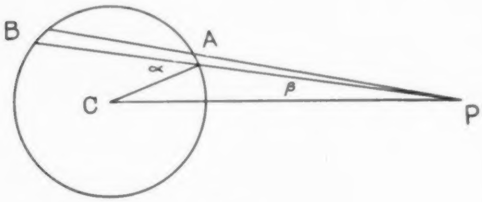


FIG. 2

According to Majorana, we must take the absorption of gravitation into account. In passing through a distance dr, this is $h\theta dr$ or hdn. Hence the transmission through the layer QA is e^{-hn}, and we find easily for the whole attraction at P, due to the frustum,

$$\frac{k}{h}(1-e^{-hn_1})d\omega .$$

Suppose now that M is spherical and that the density depends only on the distance from the center C. Let the angle CPB be β and CAB be α, also let $CP=R$, $CA=R_0$, then

$$\sin\beta = \frac{R}{R_0}\sin\alpha .$$

The whole attraction of the sphere at P must obviously be directed toward C. The component in this direction of the attraction of the frustum already considered is $kn_1 d\omega \cos\beta$. If ϕ is the position-angle about the axis CP, we have:

$$d\omega = \sin\beta\, d\beta\, d\phi = \frac{R_0^2}{R^2}\sin\alpha\cos\alpha\sec\beta\, d\alpha\, d\phi .$$

The whole attraction of the sphere at P will be

$$F = \frac{2\pi k R_0^2}{R^2}\int_0^{\frac{\pi}{2}} n_1 \sin\alpha\cos\alpha d\alpha , \tag{1}$$

according to Newton, but

$$F' = \frac{2\pi k R_0^2}{h R^2} \int_0^{\frac{\pi}{2}} (1 - e^{-hn_1}) \sin \alpha \cos \alpha d\alpha, \tag{2}$$

according to Majorana.

It is evident that n_1 depends only upon the length of the chord AB, that is, that it is a function of α, and hence that the definite integrals in (1) and (2) depend only upon the distribution of density within the sphere. Therefore the whole attraction of the sphere at an exterior point is inversely proportional to the square of the distance from the center, on Majorana's theory as well as Newton's.

When finally reduced, equation (1) goes over, of course, into the familiar form

$$F = kM/r^2 .$$

If the density is uniform, $n_1 = 2R_0\theta \cos \alpha$, and equation (2) gives Majorana's formula for ψ. For other laws of density the integrals will in general be difficult or impracticable to reduce to known forms; but if the total absorption is small, we may write:

$$\frac{1}{h}(1 - e^{-hn_1}) = n_1 - \tfrac{1}{2} h n_1^2 \; . \; . \; . \; . \; .$$

If the density θ is proportional to $(R_0^2 - S^2)$, where S is the distance from the center of the sphere, we find without difficulty that n_1 is proportional to $(R_0^2 - S_0)^{2n+\frac{1}{2}}$, where S_0 is the least distance of the chord from the center. But then $S_0 = R_0 \sin \alpha$. We may then write $n_1 = CR_0 \cos^{n+1} \alpha$ and find on Majorana's theory:

$$F' = \frac{2\pi k c R_0^3}{R^2} \left(\frac{1}{2n+3} - \frac{hcR_0}{8n+8} + \; . \; . \; . \; . \right).$$

The first term equals kM/R^2, or $4k\pi\theta_0 R_0^3/R^2$, if θ_0 is the true mean density. Hence

$$C = \frac{4N+6}{3} \theta_0,$$

and

$$F = \frac{kM}{R} \left(1 - \frac{(2n+3)^2}{12n+12} h\theta_0 R_0 \right),$$

as stated in the text.

MOUNT WILSON OBSERVATORY
July 20, 1921

NOTE

Professor D. L. Webster, in a conversation with the writer, has pointed out that the supposition of a change in velocity of a spherical body which is contracting and hence diminishing in mass (according to the alternative theory suggested in section 5) is inconsistent with the principle of relativity, since, from the magnitude of such a change (no matter whether energy or momentum should be conserved) it would be possible to determine the absolute velocity of the body.

This difficulty makes the theory of an actual change of mass appear very improbable, and the trouble is not diminished when it is asked what becomes of the energy which, on relativistic principles, is equivalent to the mass which has disappeared. Further evidence regarding the reality of the experimental effect appears to be urgently called for.

PRINCETON UNIVERSITY OBSERVATORY
September 27, 1921

INVESTIGATIONS ON PROPER MOTION

FIFTH PAPER: THE INTERNAL MOTION IN THE SPIRAL NEBULA MESSIER 81[1]

By ADRIAAN VAN MAANEN

ABSTRACT

Spiral nebula Messier 81.—Two plates taken at Mount Wilson, one by Ritchey in 1910 and the other by Duncan in 1921, were measured with the new stereocomparator, and from the shifts of 104 points in the nebula with reference to 14 comparison stars, the *proper motion of the nebula* as a whole was found to be: $\mu_\alpha = 0\overset{''}{.}014$ and $\mu_\delta = -0\overset{''}{.}005$. In addition, when the displacements of the points with reference to the nebula as a whole were corrected for the fact that the *inclination of the plane of the nebula* to the celestial sphere is about 49°, the *internal motion* was found to be, in general, a spiral motion out along the arms, of $0\overset{''}{.}039$ per year NWSE, combined with a slight outward transverse motion of about $0\overset{''}{.}007$ per year. The rotational component of the motion is $0\overset{''}{.}038$ and corresponds to a period of rotation of 58,000 years.

Formation of spiral nebulae.—The four nebulae whose internal motions have been studied, M 33, 51, 81, and 101, all have internal spiral motions such as Jeans described in his *Problems of Cosmogony and Stellar Dynamics*. This fact *suggests* that the nebulous masses were rotating and had reached a lenticular shape when the arms began to be formed by matter being thrown off at antipodal points.

In a previous paper[2] it was mentioned that measures of two photographs of M 81 taken by Mr. Ritchey in 1910 and 1916 gave preliminary evidence of internal motion, similar to that revealed in the case of M 101. Another plate of M 81 was obtained in 1921 by Mr. Duncan which made it advisable to examine the additional evidence made available by the longer interval. The photographs used for the present paper are of 1910, February 5, and 1921, April 7; both received exposures of 4 hours and 15 minutes and are of good quality. Since no plates taken with a color screen were available, the choice of nebular points and comparison stars was necessarily based on the appearance of their images. Six of the 58 points included in the former discussion on account of their coincidence with the streamers of the nebula were excluded from the present discussion in advance of measurement because of their starlike appearance; they were, however, again included in the measures. Their numbers are 4, 28, 40, 42, 51, and 54.

[1] *Contributions from the Mount Wilson Observatory*, No. 214.

[2] *Mt. Wilson Contr.*, No. 118, 17, 1916; *Astrophysical Journal*, **44**, 210, 1916.

Although the brighter stars were avoided, the comparison stars, as a whole, are somewhat brighter than the nebular points. For this reason a magnitude error may have vitiated the relative motions of the latter. While this may have affected the total motion of the nebula, it cannot have influenced the internal motion, since the magnitudes of the nebular points have a nearly symmetrical distribution around the center of the nebula. For the present investigation 14 comparison stars (the same as before) and 110 nebular points were selected for measurement.

As in the case of M 51,[1] the plates were measured with the new stereocomparator, with the directions east, west, north, and south coinciding successively with that of increasing reading of the micrometer screw.

The means of the two sets of measures in right ascension and in declination, which are expressed in parts of the micrometer screw for the interval of 11.17 years, were reduced to annual displacements in thousandths of a second of arc by multiplying them by the factor 0.0764. The resulting quantities m_α and m_δ were then reduced by the formulae:

$$\left.\begin{aligned} m_\alpha &= a+bx+cy+dx^2+cxy+fy^2+\mu_\alpha \\ m_\delta &= a'+b'x+c'y+d'x^2+c'xy+f'y^2+\mu_\delta \end{aligned}\right\} \qquad (1)$$

in which a, b, ... a', b', ... f' are the plate constants, x and y the co-ordinates of each point measured, and μ_α and μ_δ the annual proper motions, expressed in thousandths of a second of arc. For the derivation of the constants a . . . . f', the two sets of equations (1) for the comparison stars were solved by least squares, after making the usual supposition that the motions of the comparison stars are in the mean zero and are not functions of their position on the plates. Substitution of the constants thus determined into the equations (1) for all the objects measured yields the annual proper motions. These are given in the fourth and fifth columns of Table I; the second and third columns of this table give the co-ordinates of each object measured. The proper motions thus derived are relative to the group of comparison stars. To separate the motion of the nebula as a whole from the

[1] *Astrophysical Journal*, **54**, 237, 1921; *Mt. Wilson Contr.*, No. 213, 1921.

TABLE I

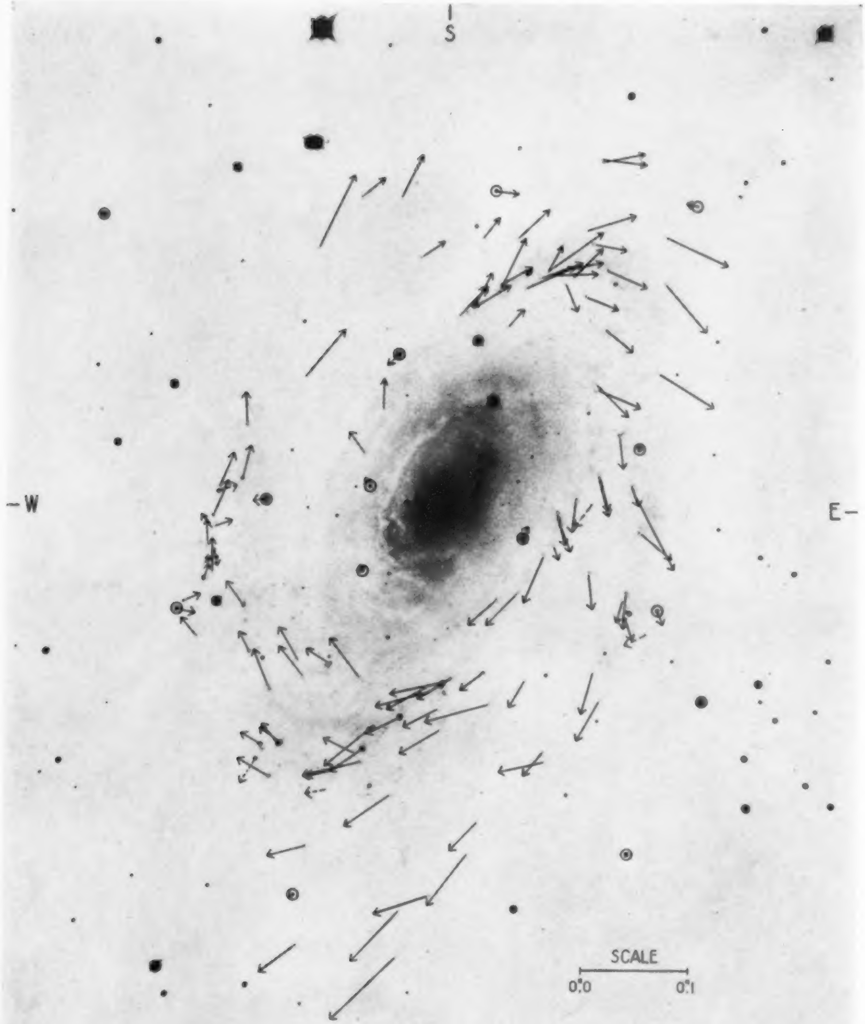
Co-ordinates and Measured Annual Displacements

No.	x	y	μ_α	μ_δ
a	+1′.7	+0′.6	+0″.005	0″.007
b	+4.6	+2.0	+ 5	+ 14
c	+5.5	+3.9	+ 2	− 1
d	+4.0	+7.1	− 2	− 2
e	−2.9	+8.0	− 5	+ 1
f	−5.5	+2.1	+ 17	+ 3
g	−1.6	+1.3	− 2	− 6
h	−3.7	−0.2	− 12	+ 1
i	−1.5	−0.5	− 5	− 8
j	−0.9	−3.2	− 10	+ 10
k	−7.1	−6.0	− 4	− 1
l	+1.0	−6.6	+ 21	+ 2
m	+5.2	−6.3	− 11	− 3
n	+4.0	−1.3	0	− 3
1	+2.5	+0.3	+ 23	+ 20
2	+2.5	0.0	+ 22	+ 26
3	+2.8	−0.3	+ 11	+ 27
4	+3.1	−0.2	− 1	+ 15
5	+3.2	−2.5	+ 52	+ 11
6	+0.3	−4.0	+ 38	− 31
7	+0.6	−4.2	+ 32	− 37
8	+0.6	−4.3	+ 43	− 22
9	+1.2	−4.7	+ 42	− 21
10	+1.7	−4.6	+ 59	− 25
11	+1.9	−4.8	+ 53	− 19
12	+2.1	−5.0	+ 27	− 31
13	+2.2	−5.1	+ 62	− 39
14	+2.2	−4.9	+ 58	− 6
15	+2.3	−5.1	+ 51	− 12
16	+2.4	−4.7	+ 25	+ 20
17	+2.7	−4.9	+ 40	− 30
18	+3.0	−5.6	+ 45	0
19	+2.9	−5.9	+ 60	− 20
20	+3.4	−5.1	+ 51	+ 11
21	+4.5	−4.8	+ 52	+ 40
22	+3.9	−0.6	+ 20	+ 14
23	+4.0	−0.5	+ 52	+ 67
24	+4.1	+0.4	+ 43	+ 20
25	+3.1	+1.2	+ 16	+ 31
26	+3.9	+1.7	+ 12	+ 29
27	+3.9	+2.2	+ 20	+ 14
28	+4.3	+2.5	− 5	+ 5
29	+2.2	+5.0	− 4	+ 17
30	−0.2	+7.9	− 37	+ 12
31	−2.9	+9.0	− 28	+ 19
32	+1.2	+1.8	− 12	+ 19
33	+1.0	+3.3	− 7	+ 11
34	+0.3	+3.5	− 16	+ 14
35	+0.1	+3.6	− 34	+ 5
36	−0.3	+3.7	− 22	+ 15
37	−0.2	+3.8	− 35	+ 9
38	−0.8	+4.2	− 16	+ 10

TABLE I—*Continued*

No.	x	y	μ_α	μ_δ
39	−1′.6	+5′.0	−0″.016	+0″.015
40	−2.3	+5.8	− 4	− 2
41	−3.5	+5.5	− 15	− 22
42	−3.8	+5.2	0	+ 17
43	−3.7	+4.9	− 5	− 16
44	−3.3	+4.9	− 3	− 24
45	−3.2	+4.8	− 5	− 19
46	−4.0	+2.9	+ 5	− 24
47	−4.0	+2.0	− 2	− 28
48	−5.3	+1.9	+ 29	− 12
49	−4.9	+1.5	+ 19	− 19
50	−4.8	+1.3	+ 14	− 38
51	−4.7	+0.8	+ 16	+ 10
52	−4.8	+0.6	+ 13	− 25
53	−4.7	+0.1	+ 28	− 37
54	−4.6	−0.4	+ 30	− 11
55	−4.1	−0.6	+ 22	− 35
56	−1.7	−6.5	+ 36	− 22
57	−0.5	−5.2	+ 36	− 20
58	−1.6	−1.2	0	− 24
59	+2.2	+0.8	− 4	+ 39
60	+2.4	+0.6	+ 19	+ 8
61	+3.3	−0.7	+ 21	+ 27
62	+3.2	−0.8	+ 23	+ 34
63	+3.8	+1.7	+ 8	+ 17
64	+3.2	+3.3	+ 4	+ 35
65	+3.4	+3.9	− 7	+ 32
66	+2.3	+5.2	− 29	+ 4
67	+0.8	+6.3	− 11	+ 21
68	+0.6	+7.1	− 23	+ 44
69	−0.8	+8.3	− 30	+ 42
70	−0.8	+9.2	− 45	+ 52
71	−2.8	+6.9	− 21	+ 4
72	−2.1	+5.3	− 10	+ 1
73	−1.0	+5.9	− 27	+ 23
74	−1.7	+5.2	− 38	+ 8
75	−1.7	+5.0	− 14	− 18
76	−1.4	+4.5	− 30	+ 32
77	0.0	+4.0	− 17	+ 11
78	0.0	+4.5	− 23	+ 17
79	+1.0	+4.0	− 44	+ 11
80	+1.8	+3.4	0	+ 21
81	+1.7	+1.7	− 15	+ 25
82	−2.2	+3.2	− 9	− 19
83	−2.9	+3.1	− 4	− 36
84	−2.9	+3.4	− 8	− 32
85	−3.5	+3.7	− 1	− 42
86	−5.1	+2.7	0	− 22
87	−4.8	+1.0	+ 18	− 5
88	−4.9	+1.1	+ 25	− 11
89	−4.7	+0.4	+ 4	− 18
90	−4.6	+0.4	+ 30	− 11
91	−4.6	−0.5	+ 27	− 36
92	−4.0	−1.7	+ 12	− 36

PLATE IV



INTERNAL MOTIONS IN MESSIER 81

The arrows indicate the direction and magnitude of the mean annual motions. Their scale (0.″1) is indicated on the illustration. The scale of the nebula is 1 mm = 9.″3. The comparison stars are inclosed in circles.

TABLE I—*Continued*

No.	x	y	μ_α	μ_δ
93	$-2'.9$	$-2'.8$	$+0''.051$	$-0''.050$
94	-2.7	-5.4	$+\ 47$	$-\ 73$
95	-0.9	-6.5	$+\ 33$	$-\ 45$
96	-1.2	-2.0	$+\ 14$	$-\ 32$
97	$+0.6$	-4.3	$+\ 25$	$-\ 20$
98	$+0.7$	-5.7	$+\ 29$	$-\ 24$
99	$+1.2$	-4.6	$+\ 36$	$-\ 52$
100	$+1.3$	-3.8	$+\ 30$	$-\ 23$
101	$+1.5$	-5.7	$+\ 43$	$-\ 32$
102	$+2.9$	-4.5	$+\ 44$	$+\ 6$
103	$+4.6$	-5.7	$+\ 74$	$+\ 22$
104	$+3.4$	-3.8	$+\ 39$	$+\ 16$
105	$+3.2$	-2.6	$+\ 44$	$+\ 25$
106	$+3.7$	-1.6	$+\ 18$	$+\ 26$
107	$+4.5$	-2.9	$+\ 66$	$+\ 27$
108	$+3.1$	-7.4	$+\ 57$	0
109	$+3.2$	-7.3	$+\ 55$	$-\ 13$
110	-1.7	$+3.4$	$-\ 13$	$-\ 40$

internal motion, the same process was used as previously in the case of M 101 and M 51, with the following results:

a) The mean motion of the 104 nebular points (excluding Nos. 4, 28, 40, 42, 51, and 54 for reasons given on page 347) is $\mu_\alpha = +0''.014$, $\mu_\delta = -0''.002$.

b) Combining the mean motions in quadrants I and III, we have $\mu_\alpha = +0''.017^5$, $\mu_\delta = -0''.005$; while for quadrants II and IV, $\mu_\alpha = +0''.012^5$, $\mu_\delta = -0''.008$. All four quadrants combined give $\mu_\alpha = +0''.015$, $\mu_\delta = -0''.006^5$.

c) Excluding points outside a circle with radius $5'.5$, we find for quadrants I and III $\mu_\alpha = +0''.017$, $\mu_\delta = -0''.006$; for quadrants II and IV $\mu_\alpha = +0''.011^5$, $\mu_\delta = -0''.006$. For all four quadrants $\mu_\alpha = +0''.014$, $\mu_\delta = -0''.006$.

For the motion of the nebula as a whole the mean result from the three methods is

$$\mu_\alpha = +0''.014, \qquad \mu_\delta = -0''.005.$$

Subtracting these values from the annual motions given in Table I, we derive the internal motions, which are given in the second and third columns of Table II. These motions are plotted in Plate IV. For the comparison stars, which are surrounded by circles, the

TABLE II

ANNUAL INTERNAL MOTIONS

No.	μ_α	μ_δ	Rotational	Radial	Stream	Transverse
1	+0".009	+0".025	+0".024	+0".025	+0".017	+0".030
2	+ 8	+ 31	+ 32	+ 25	+ 25	+ 32
3	− 3	+ 32	+ 35	+ 6	+ 33	+ 12
4	− 15	+ 20	....	....	....	....
5	+ 38	+ 16	+ 35	+ 31	+ 15	+ 45
6	+ 24	− 26	+ 30	+ 21	+ 36	+ 8
7	+ 18	− 32	+ 17	+ 34	+ 28	+ 26
8	+ 29	− 17	+ 35	+ 14	+ 38	+ 3
9	+ 28	− 16	+ 34	+ 15	+ 37	+ 6
10	+ 45	− 20	+ 57	+ 21	+ 57	+ 20
11	+ 39	− 14	+ 48	+ 20	+ 50	+ 14
12	+ 13	− 26	+ 2	+ 30	+ 4	+ 30
13	+ 48	− 34	+ 45	+ 45	+ 50	+ 40
14	+ 44	− 1	+ 59	+ 12	+ 60	+ 3
15	+ 37	− 7	+ 50	+ 13	+ 51	+ 2
16	+ 11	+ 25	+ 35	− 18	+ 33	− 20
17	+ 26	− 25	+ 16	+ 34	+ 14	+ 35
18	+ 31	+ 5	+ 45	+ 9	+ 45	+ 4
19	+ 46	− 15	+ 52	+ 31	+ 55	+ 22
20	+ 37	+ 16	+ 58	+ 10	+ 59	0
21	+ 38	+ 45	+ 85	+ 10	+ 85	+ 8
22	+ 6	+ 19	+ 22	+ 14	+ 25	+ 10
23	+ 38	+ 72	+ 82	+ 76	+ 91	+ 65
24	+ 29	+ 25	+ 23	+ 53	+ 30	+ 50
25	+ 2	+ 36	+ 33	+ 38	+ 32	+ 39
26	− 2	+ 34	+ 32	+ 21	+ 35	+ 14
27	+ 6	+ 19	+ 14	+ 23	+ 18	+ 19
28	− 19	+ 10	....	....	....	....
29	− 18	+ 22	+ 28	+ 8	+ 29	+ 1
30	− 51	+ 17	+ 63	− 2	+ 55	− 31
31	− 42	+ 24	+ 46	+ 28	+ 54	− 6
32	− 26	+ 24	+ 35	+ 1	+ 35	− 3
33	− 21	+ 16	+ 27	+ 3	+ 27	− 4
34	− 30	+ 19	+ 38	+ 5	+ 37	− 13
35	− 48	+ 10	+ 61	− 11	+ 52	− 34
36	− 36	+ 20	+ 45	+ 10	+ 45	− 9
37	− 49	+ 14	+ 63	− 2	+ 58	− 25
38	− 30	+ 15	+ 36	+ 12	+ 37	− 7
39	− 30	+ 20	+ 32	+ 22	+ 39	0
40	− 18	+ 3	....	....	....	....
41	− 29	− 17	+ 51	+ 1	+ 51	0
42	− 14	+ 22	....	....	....	....
43	− 19	− 11	+ 33	+ 4	+ 33	+ 4
44	− 17	− 19	+ 36	− 6	+ 36	− 2
45	− 19	− 14	+ 36	+ 1	+ 36	0
46	− 9	− 19	+ 25	− 3	+ 25	− 4
47	− 16	− 23	+ 34	+ 16	+ 35	+ 13
48	+ 15	− 7	+ 2	− 19	0	− 19
49	+ 5	− 14	+ 14	− 6	+ 13	− 8

TABLE II—*Continued*

No.	μ_α	μ_δ	Rotational	Radial	Stream	Transverse
50	0".000	−0".033	+0".036	+0".002	+0".036	−0".002
51	+ 2	+ 15	....	....	....	....
52	− 1	− 20	+ 24	+ 6	+ 24	+ 3
53	+ 14	− 32	+ 35	− 8	+ 32	− 15
54	+ 16	− 6	....	....	....	....
55	+ 8	− 30	+ 35	+ 5	+ 35	− 4
56	+ 22	− 17	+ 30	+ 3	+ 30	− 3
57	+ 22	− 15	+ 29	+ 3	+ 29	− 2
58	− 14	− 19	+ 7	+ 31	+ 12	+ 30
59	− 18	+ 44	+ 45	+ 5	+ 44	+ 10
60	+ 5	+ 13	+ 11	+ 14	+ 8	+ 15
61	+ 7	+ 32	+ 44	+ 20	+ 33	+ 36
62	+ 9	+ 39	+ 39	+ 17	+ 29	+ 31
63	− 6	+ 22	+ 26	+ 8	+ 27	+ 5
64	− 10	+ 40	+ 35	+ 27	+ 38	+ 22
65	− 21	+ 37	+ 40	+ 13	+ 40	+ 11
66	− 43	+ 9	+ 48	− 28	+ 38	− 41
67	− 25	+ 26	+ 36	+ 19	+ 39	+ 7
68	− 37	+ 49	+ 49	+ 37	+ 60	+ 13
69	− 44	+ 47	+ 50	+ 41	+ 64	+ 6
70	− 59	+ 57	+ 68	+ 48	+ 82	+ 9
71	− 35	+ 9	+ 41	+ 15	+ 38	− 20
72	− 24	+ 6	+ 28	+ 9	+ 30	+ 4
73	− 41	+ 28	+ 48	+ 23	+ 50	− 17
74	− 52	+ 13	+ 64	+ 16	+ 65	− 3
75	− 28	− 13	+ 43	− 12	+ 38	− 25
76	− 44	+ 37	+ 44	+ 41	+ 55	+ 25
77	− 31	+ 16	+ 39	+ 7	+ 37	− 15
78	− 37	+ 22	+ 47	+ 8	+ 43	− 19
79	− 58	+ 16	+ 69	− 26	+ 62	− 40
80	− 14	+ 26	+ 26	+ 15	+ 28	+ 9
81	− 29	+ 30	+ 43	− 2	+ 42	− 8
82	− 23	− 14	+ 40	+ 4	+ 40	+ 1
83	− 18	− 31	+ 49	− 4	+ 49	− 7
84	− 22	− 27	+ 49	− 4	+ 49	− 6
85	− 15	− 37	+ 51	− 10	+ 52	− 5
86	− 14	− 17	+ 29	+ 13	+ 28	+ 15
87	+ 4	0	− 2	− 6	− 2	− 6
88	+ 11	− 6	+ 2	− 14	+ 1	− 14
89	− 10	− 13	+ 16	+ 16	+ 16	+ 16
90	+ 16	− 6	+ 5	− 21	+ 2	− 21
91	+ 13	− 31	+ 34	− 4	+ 32	− 13
92	− 2	− 31	+ 28	+ 23	+ 32	+ 17
93	+ 37	− 45	+ 58	− 4	+ 58	− 9
94	+ 33	− 68	+ 64	+ 40	+ 71	+ 25
95	+ 19	− 40	+ 31	+ 33	+ 34	+ 29
96	0	− 27	+ 16	+ 28	+ 25	+ 20
97	+ 11	− 15	+ 12	+ 17	+ 19	+ 7
98	+ 15	− 19	+ 17	+ 16	+ 18	+ 15
99	+ 22	− 47	+ 13	+ 51	+ 35	+ 40
100	+ 16	− 18	+ 13	+ 21	+ 20	+ 15

TABLE II—*Continued*

No.	μ_a	μ_δ	Rotational	Radial	Stream	Transverse
101	+0".029	−0".027	+0".032	+0".028	+0".033	+0".026
102	+ 30	+ 11	+ 46	+ 9	+ 45	+ 14
103	+ 60	+ 27	+ 94	+ 28	+ 95	+ 21
104	+ 25	+ 21	+ 48	+ 8	+ 39	+ 28
105	+ 30	+ 30	+ 59	+ 23	+ 45	+ 44
106	+ 4	+ 31	+ 36	+ 6	+ 34	+ 14
107	+ 52	+ 32	+ 70	+ 53	+ 62	+ 62
108	+ 43	+ 5	+ 62	+ 5	+ 62	− 6
109	+ 41	− 8	+ 50	+ 17	+ 51	+ 8
110	− 27	− 35	+ 57	− 30	+ 62	− 18

motions of Table I are used. The scale of the motions is indicated in the lower right-hand corner. The arrows represent the motions during an interval of about 1,300 years.

As in the case of M 101, 33, and 51, there is a question as to whether the displacements are best represented by a rotation or by a motion along the arms of the spiral. To investigate this, the motions were resolved into (*a*) rotational and radial components; (*b*) components along and perpendicular to the arms of the spirals (stream and transverse components). In order to do this, a diagram was made in which the foreshortening of the nebula was corrected. This was necessary because M 81 is decidedly elliptical in appearance, due undoubtedly to an inclination of the plane of the nebula to the tangential plane of the celestial sphere. It was estimated that the intersection of the two planes is in position angle 149° and that the inclination is 49°. Hence, the positions and components of the measured objects in the direction of position angles 149° and 329° were made the same in the diagram as on the photographs, while distances and components at right angles to this direction were increased in the ratio $1/\cos 49° = 1.5$. The results for individual points derived from the diagram are in the last four columns of Table II, where, as before, the positive sign is used for motions in the directions north, west, south, east, and outward.

The mean rotational component is +0".038; the mean radial component, +0".013, which is 34 per cent of the rotational vector. Since the mean distance of the nebular points from the center is

5′.8, the rotational component would correspond to a hypothetical period of about 60,000 years. The mean stream component is +0″.039, with a transverse component of +0″.007.

The evidence on internal motions derived from the four nebulae which have been measured, viz., M 101, 33, 51, and 81, is summarized briefly in Table III, in which the second column gives the focal length used, the third column the interval in years. The next four columns give the mean rotational, radial, stream, and transverse components of the motions derived, while the last column gives the number of nebular points measured.

TABLE III

SUMMARY OF INTERNAL MOTIONS IN SPIRAL NEBULAE

(Unit for motions and their probable errors is 0″.001)

Object	Focus in Feet	Interval in Years	Rotational	Radial	Stream	Transverse	*n*
M 101*	25	5	+21±1	+ 5±1	+21±1	0±1	87
M 101†	18	9	+20±2	+ 6±2	+22±2	− 3±2	69
M 101†	18	15	+12±2	+ 7±2	+14±2	+ 2±2	46
M 33	25	10	+20±3	+ 6±2	+24±3	− 2±2	30
M 33	80	5	+14±4	+12±3	+18±4	+ 4±3	21
M 51	25	11	+19±1	+ 8±1	+21±1	+ 3±1	79
M 81	25	6	+20±4	+17±3	+25±3	+16±3	52
M 81	25	11	+38±1	+13±1	+39±1	+ 7±1	104

* Including the measures by Mr. Nicholson, *Mt. Wilson Contr.*, No. 118, 18, 1916.

† Photographs taken with the Crossley reflector of the Lick Observatory. All others were taken with the 60-inch reflector at Mount Wilson.

All pairs of plates show the same type of motion, and when we keep in mind the different numbers of points measured, the agreement in the values of the motion for each nebula derived from different pairs of plates is as satisfactory as could be expected.

The rotational components would correspond to the following periods: for M 101, 85,000 years; for M 33, 160,000 years; for M 51, 45,000 years; for M 81, 58,000 years.

For M 101 there is no appreciable change in the measured rotational component with distance from the center, but for M 33, 51, and 81 there appears to be some increase of motion with distance.

All four nebulae show a large outward radial component, which for M 101 is 32 per cent of the rotational component; for M 33, 48 per cent; for M 51, 42 per cent; for M 81, 34 per cent.

The transverse components are in general very small and well within the limits of error, although the positive value for M 81 may be real. In all cases, however, the measured displacements agree better with the hypothesis of outward motion along the arms of the spiral than they do with an assumed rotation of the nebulae as a whole.

The close agreement of the displacements in direction with the spiral arms suggests that we have here a realization of the motions described by Jeans in *Problems of Cosmogony and Stellar Dynamics*, from which I quote freely.

If so, we must then suppose that before the formation of the spiral arms the nebular masses were rotating and had reached a lenticular shape. The formation of Laplace's ring requires perfect symmetry of mass about the axis of rotation. The distances of adjacent masses in space are in general so great that their gravitational influence will be extremely small, but even the slightest external gravitational field will be sufficient to preclude the formation of a ring; instead of this, the matter will be thrown off at two antipodal points. The masses first thrown off at the antipodal points form in themselves a tide-generating system which concentrates the region of ejection more and more at two points. The result is an extension farther and farther into the equatorial plane as the evolution of the nebula proceeds. The determination of the shape of the arms seems at present to be beyond the reach of mathematical analysis, but the long streams of gaseous matter must become longitudinally unstable and will tend to break up into condensations or nuclei.

Mount Wilson Observatory
July 1921

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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

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ANALYTIC INDEX OF VOLUMES 51-54
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BASED ON THE ANALYTIC ABSTRACTS

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ANALYTIC INDEX OF *ASTROPHYSICAL JOURNAL* FOR 1920 AND 1921, VOLUMES **51–54**, BASED ON THE ANALYTIC ABSTRACTS

INTRODUCTION

For two years analytic abstracts have been accompanying the articles in the *Astrophysical Journal*. The use of italicized subtitles has emphasized the fact that most titles do not describe the contents completely and precisely. Evidently an index of the titles, such as that provided by the *Journal*, however well made, cannot be a complete and precise index of the contents; and since in connection with the abstracts subtitles have been prepared which aim to be both complete and precise, why not form an index of them? This has in fact been done; and the index entries have been thoroughly classified so as to get all the references to each particular subject, like X-rays and Zeeman effect, together. It is hoped that the following analytic index made in this way will not only prove of value to the readers of the *Astrophysical Journal* but will also recommend itself so highly that other journals will provide such indexes and especially that our abstract journals will do so. The number of words in the following index is only twice the number in the corresponding title indexes, and while the labor of making it is of course considerably greater, it need not be excessive if properly organized, and no unusual ability is required on the part of the indexer.

INDEX

For references to spectra and spectroscopy of elements and compounds *see* Spectra.

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